Theory of Communications Systems

Prof. Ing. Anton Čižmár, CSc.

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Prednášky:

a) Súbory sú alebo budú postupne vkladané na web stránku predmetu TKS
b) http://kemt.fei.tuke.sk/tts/teoria-telekomunikacnych-systemov-o-predmete/
c) https://tks.kemt.website.tuke.sk/
d) Je dobré mať počas prednášky poznámkový blok na doplňujúce informácie prednášajúceho
e) Súbory sú v anglickom jazyku, prednášky v slovenskom jazyku
f) Sledovať oznamy na web stránke predmetu

Cvičenia:

a) 1 písomná skúška uprostred semestra (max 20 bodov)
b) 1 tímové klasifikované zadanie v závere semestra (max 20 bodov)

Bodové rozdelenie:

a) max. 40 bodov cvičenia – minimálne 21 bodov k zápočtu
b) max. 60 bodov za skúšku – minimálne 31 bodov k úspešnému absolvovaniu skúšky
(z toho 10 bodov za: účasť na minimálne 10 prednáškach a 2 otázky počas prednášok)

WHY this course?
WHY Theory of Communication Systems?

if

AI – Artificial Intelligence

is able to find a solution
is able to solve any problem

BUT

Are you able to exactly define a problem?

Are you able to await at least very unclear solution?

!!!

Image you have no background and basic knowledge of the field

???

This is WHY this course!
This is WHY Theory of Communication Systems!

What is the issue.
What is the problem.
What is he aim of our lectures.

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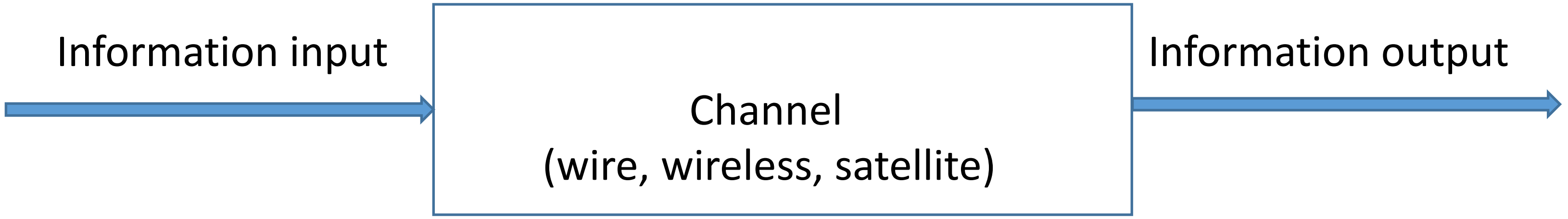
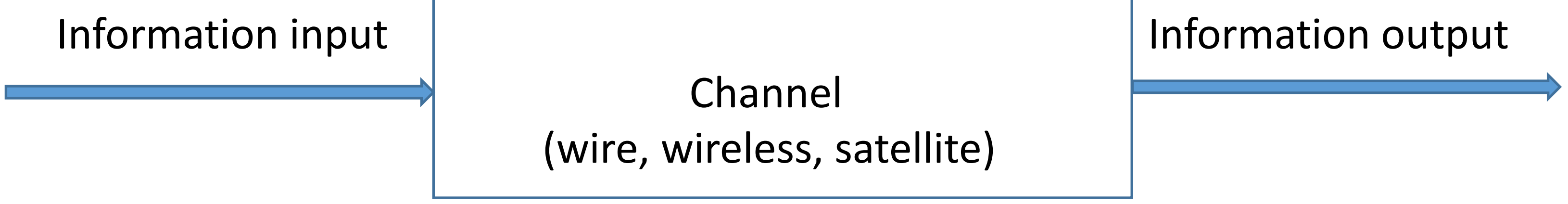


To achieve the most effective transmission of information

What is the issue.
What is the problem.
What is he aim of our lectures.



To achieve the most effective transmission of information

What kind of instruments we can use o achieve the goal

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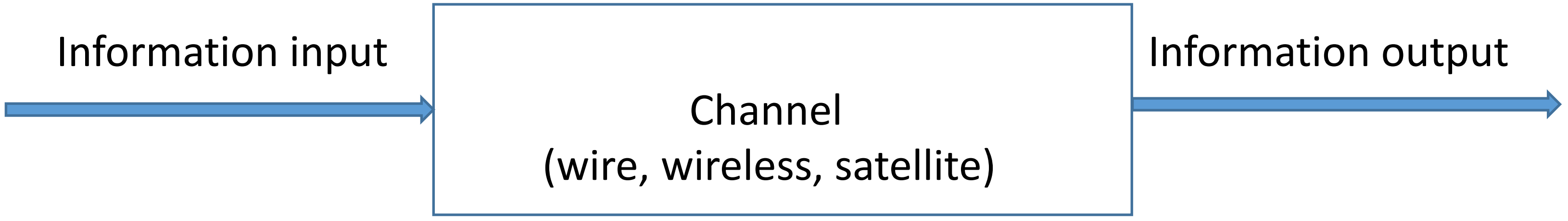


To achieve the most effective transmission of information

What kind of instruments we can use to achieve the goal

This is the agenda of our lectures – Theory of Communication Systems

The aim of a digital communication system is
to transmit the message efficiently over the communication channel (AWGN) by incorporating

- various data compressions (source coding),
- encoding (channel coding) and
- modulation techniques,

in order to reproduce the message in the receiver with the least errors.

Perhaps the greatest contribution from the math center was the creation of Information Theory by Claude Shannon in 1948.

For perhaps the first 25 years of its existence, Information Theory was regarded as a beautiful theory but not as a central guide to the architecture and design of communication systems.

After that time, however, both the device technology and the engineering understanding of the theory were sufficient to enable system development to follow information theoretic principles.

Fifth generation wireless (5G) - is a wireless networking architecture

5G x100

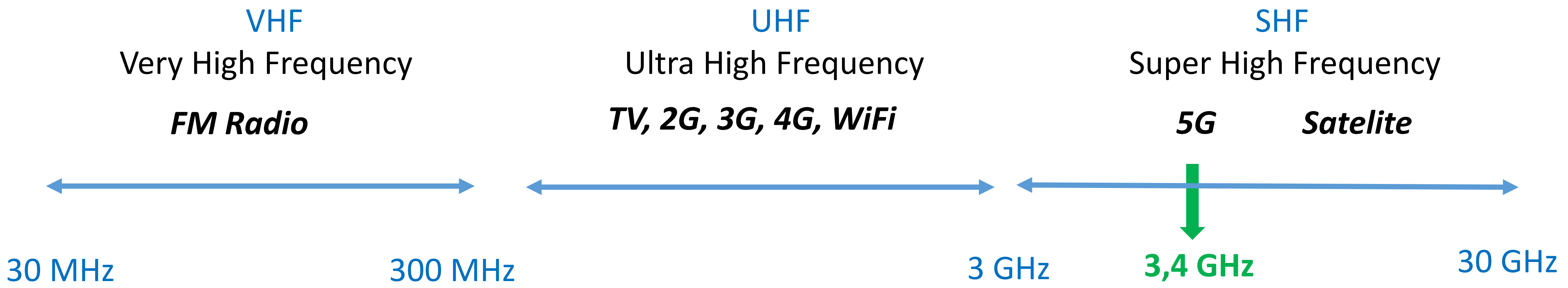
Capacity than 4G

Downloading a film

3G	4G	5G
26 hours	6 min	3,6 sec

Response time

4G	5G
0,045 sec	0,001 sec

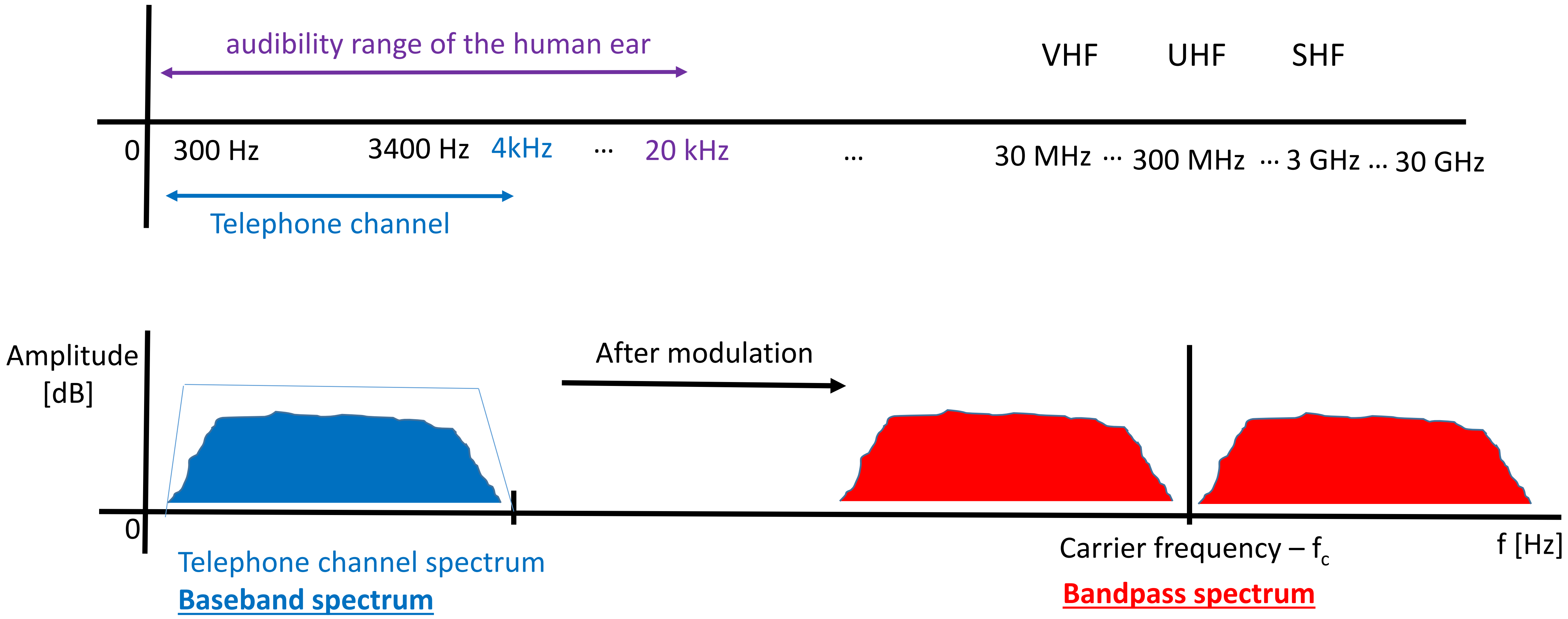


One of the key issues with any form of 5G modulation scheme is
the spectral efficiency.

With spectrum being at a premium, especially in frequencies around 3 GHz,
it is essential that any modulation scheme adopted for 5G is able **to provide a high level of spectral efficiency**.

There is often a **balance** between **higher orders of modulation** like 64 QAM as opposed to 16 QAM for example
and **noise performance**.
Thus higher order modulation schemes tend to be only sued when there is a **good signal to noise ratio**.

Some Specialities of Frequency Spectrum



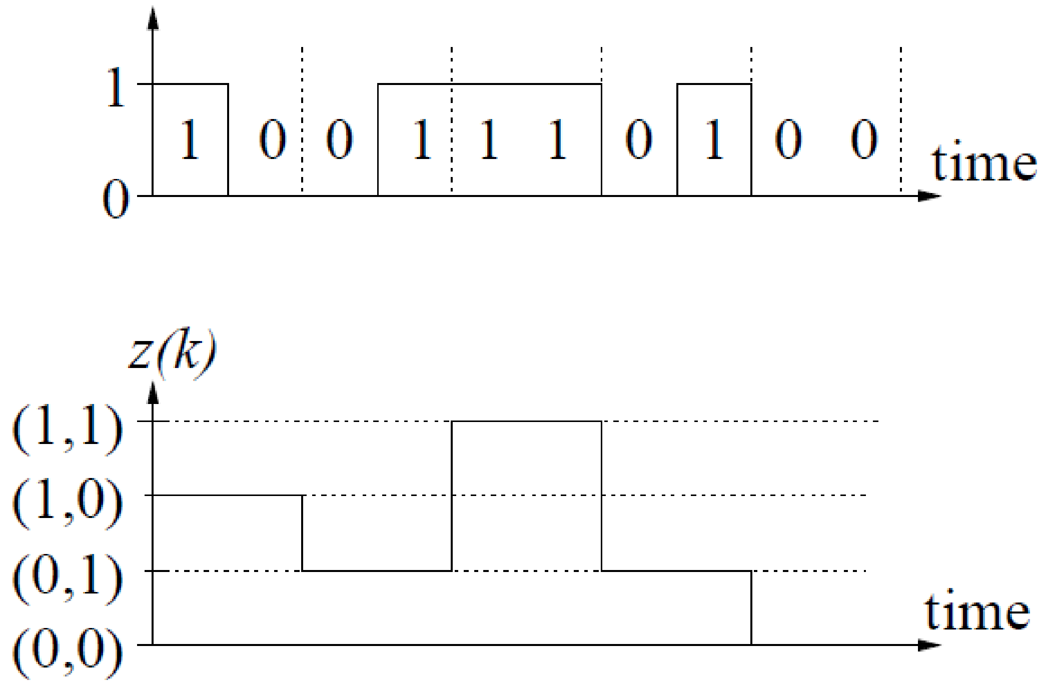
Bits to Symbols

- The bit stream to be transmitted is serial to parallel multiplexed onto a stream of symbols with q bits per symbol (discrete 2^q levels)

- Example for $q = 2$ bits per symbol (4-ary modulation): symbol period T_s is twice of bit period T_b

bit stream

symbol stream



$V_m = 1/T_s$ [Baud, Bd]
modulacna rychlost
(sirka pasma Hz, ***designer***!)

$V_p = V_m \log_2 M$ [bps]
prenosova rychlost
(rychlost prenosu, ***uzivatel***!)

M – pocet stavov (symbolov)
T_s - dlzka symbol

ak M =2; $V_m = V_p$

- Symbol rate is half of bit rate; symbol stream is then pulse shaped ... (what happens to required bandwidth?)

Bits to Symbols

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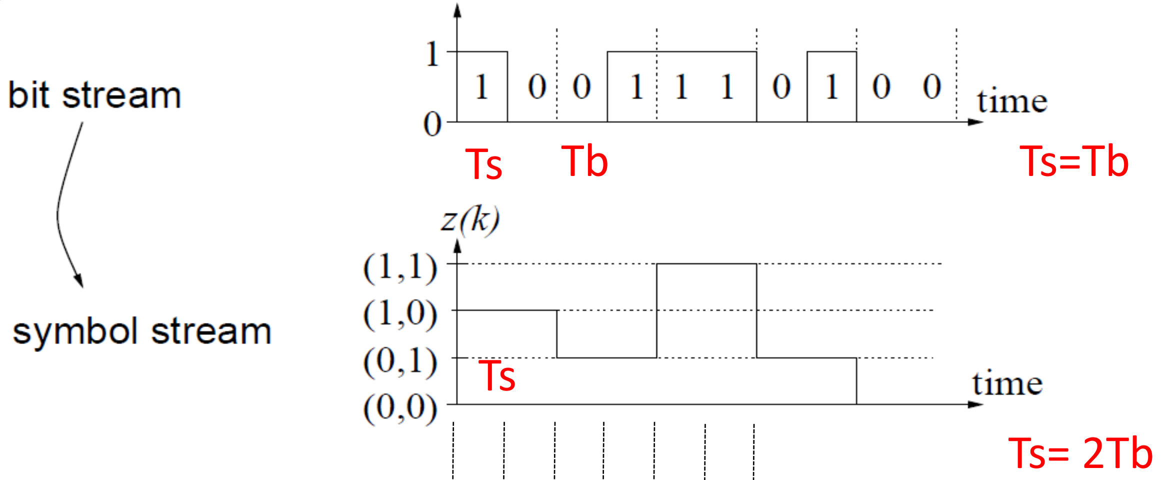
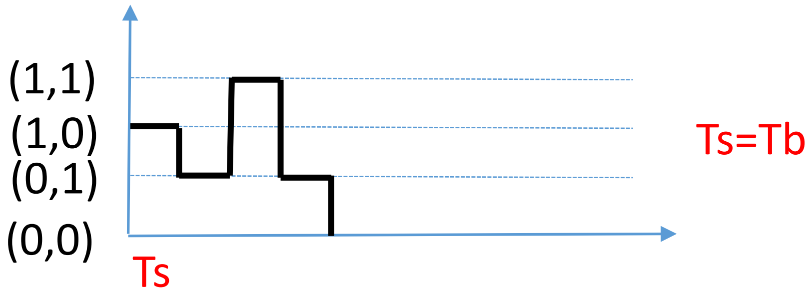


$V_m = 1/T_s$ $V_p = V_m \log_2 2$ $V_p = V_m$

$V_m = 1/2T_b$ $V_p = V_m \log_2 4$ $V_p = V_m$

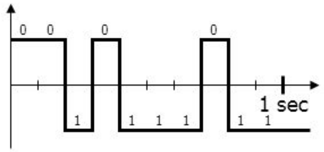
- Symbol rate is half of bit rate; symbol stream is then pulse shaped ... (what happens to required bandwidth?)



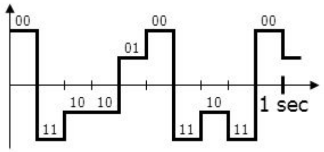
$V_m = 1/T_s$ $V_p = V_m \log_2 4$ $V_p = 2V_m$

Baud and Bit Rate

- **Baud** → How many times a signal changes per second
- **Bit rate** → How many bits can be sent per time unit (usually per second)
- Bit rate is controlled by baud and number of signal levels



Baud = 10
Bit rate = 10 bps



Baud = 10
Bit rate = 20 bps

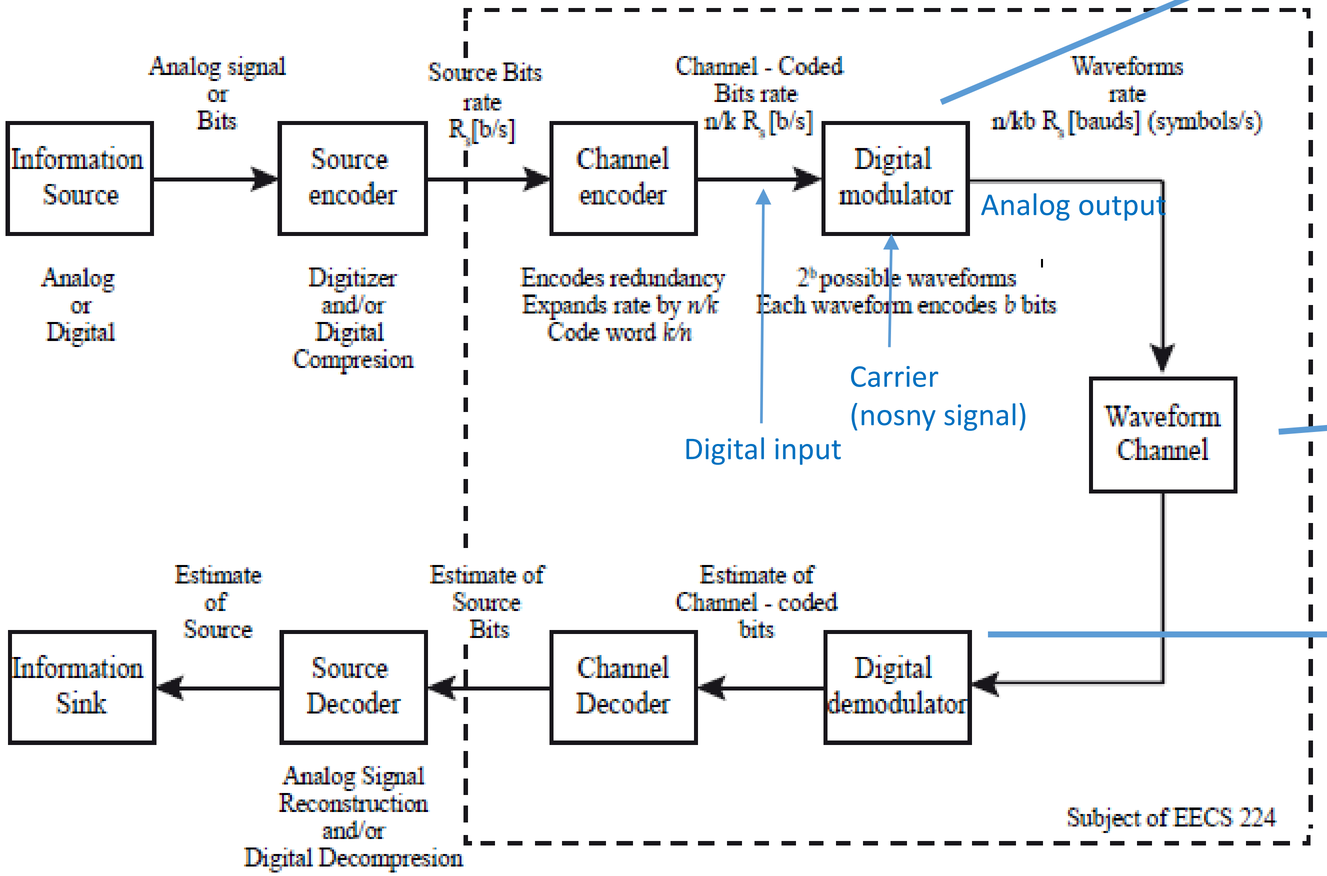


Figure 1.1 Basic elements of a digital communication system

- Interface to the communication channel
- To map binary info sequence into signal waveform
- Binary modulation $0 \rightarrow s_0(t), 1 \rightarrow s_1(t)$
- M-ary modulation $M=2^b$ distinct waveforms $s_i(t)$,

- The physical medium
- Additive (thermal) noise

AWGN channel (Additive White Gaussian Noise)

- Average probability of a bit-error at the output of the decoder

What's Modulation & Demodulation?

- **Digital modulation and demodulation:**
  - Modulation (demodulation) maps (retrieves) the digital information into (from) an analog waveform *appropriate for transmission over the channel*.
  - Generally involve translating (recovering) the baseband digital information to (from) a bandpass analog signal at a *carrier frequency* that is very high compared to the baseband frequency.
  - Examples: ASK, FSK, QPSK, 16QAM

Why Carrier?

- *Effective radiation of electromagnetic waves* requires antenna dimensions comparable with the signal's wavelength:
  - Antenna for 3 kHz carrier would be ~100 km long
  - Antenna for 3 GHz carrier is 10 cm long
- *Frequency division multiplexing*
  - Shifting the baseband signals to different carrier frequencies
  - Sharing the communication channel resources

Signal wavelength

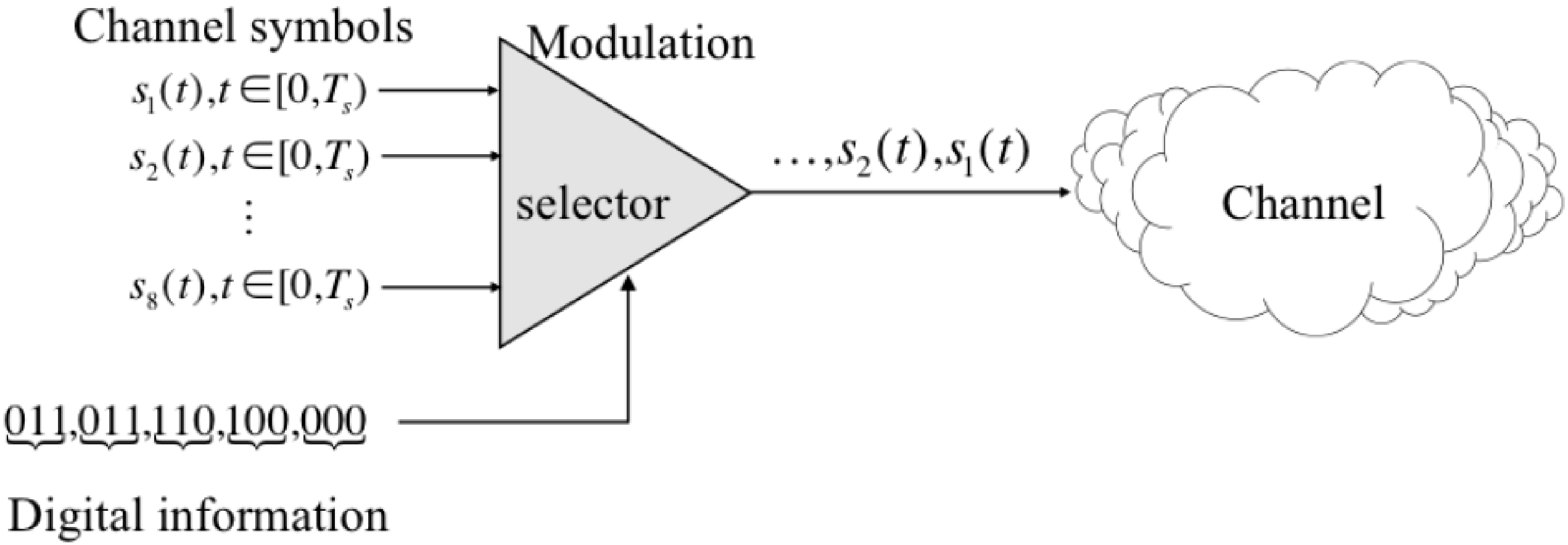
$$\lambda = \frac{c}{f} = \frac{light\ speed}{frequency} = \frac{3.10^8}{f}\ \frac{[\frac{m}{s}]}{[Hz = 1/s]}$$

Antenna for 3 kHz carrier would be

$$\lambda = \frac{c}{f} = \frac{3.10^8}{3.10^3} = 1.10^5[m] = 100\ km$$

Antenna for 3 GHz carrier would be

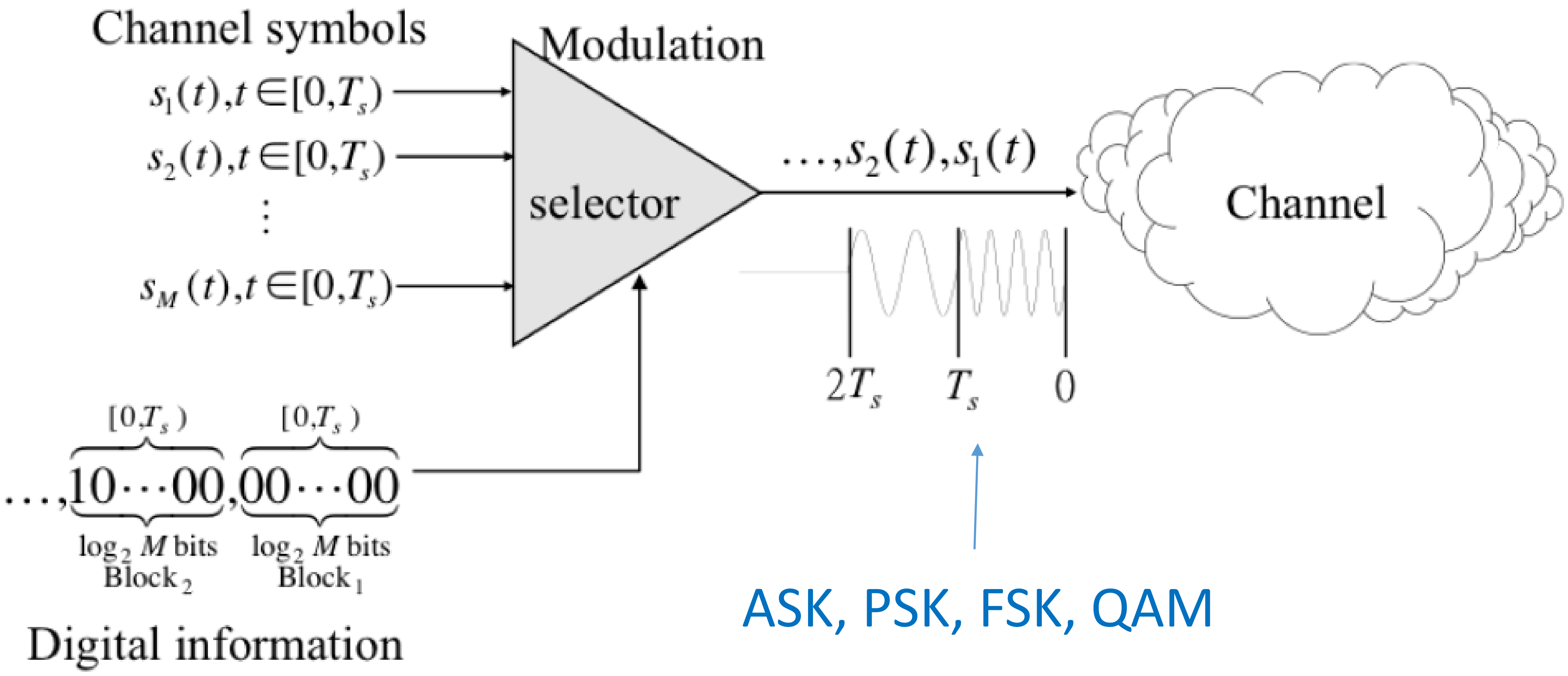
$$\lambda = \frac{c}{f} = \frac{3.10^8}{3.10^9} = 1.10^{-1}[m] = 10\ cm$$



Note that the channel symbols are bandpass signals.

Basic Modulation

- Modulation involves operations on one or more of the three characteristics of a carrier signal: ***amplitude, frequency and phase***.
- The three basic modulation methods are:
  - *Amplitude Shift Keying (ASK)*
  - *Phase Shift Keying (PSK)*
  - *Frequency Shift Keying (FSK)*
- These could be applied to binary or M-ary signals.
- There are other variants as well.

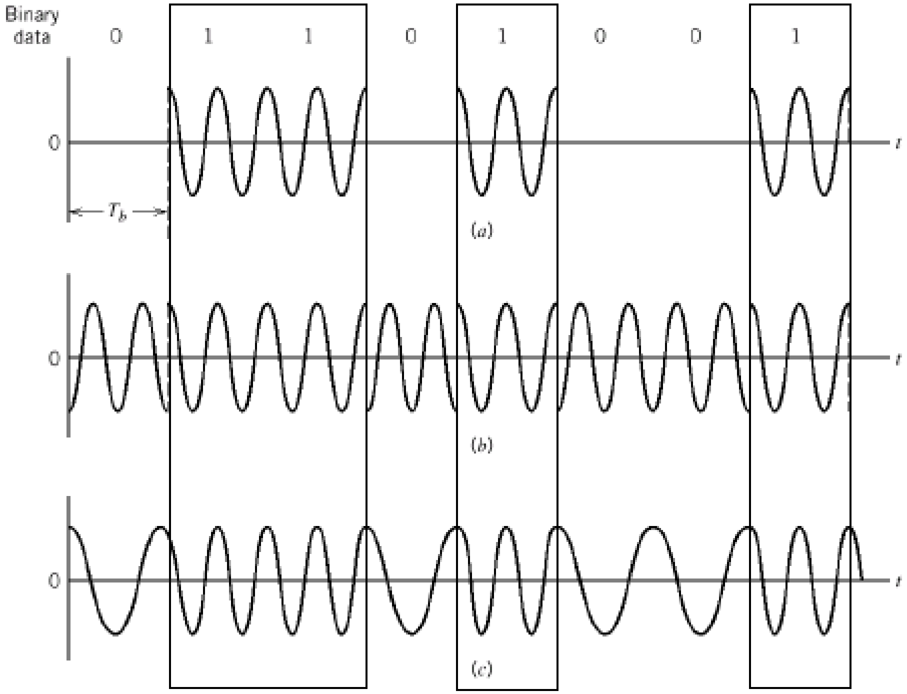

Channel symbols
$s_1(t), t \in [0, T_s)$
$s_2(t), t \in [0, T_s)$
$s_M(t), t \in [0, T_s)$
Modulation
selector
$\ldots, s_2(t), s_1(t)$
Channel
$2T_s$
T_s
0
$[0, T_s)$
$[0, T_s)$
$\ldots, 10\cdots00, 00\cdots00$
$\log_2 M$ bits
Block$_2$
$\log_2 M$ bits
Block$_1$
Digital information
ASK, PSK, FSK, QAM

- Three basic signaling schemes in digital communications

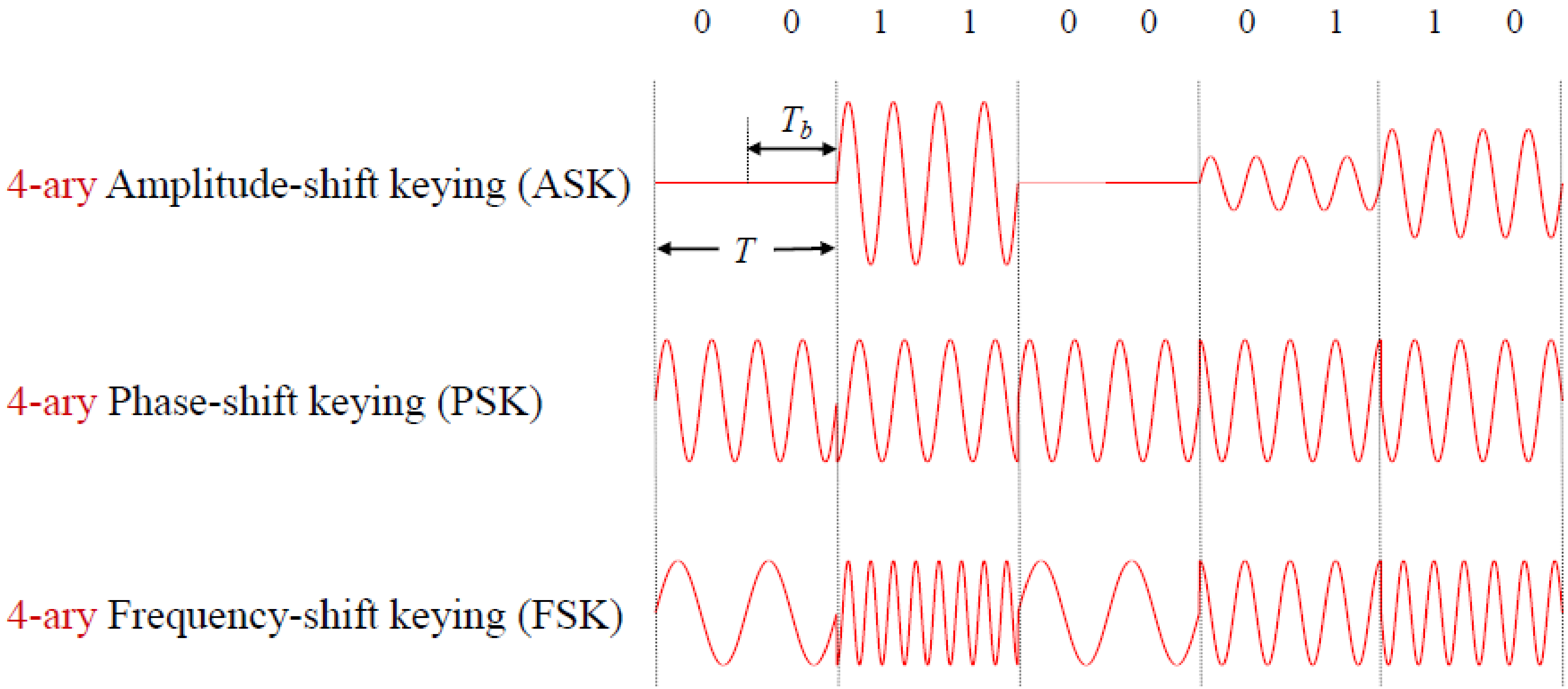
Amplitude-shift keying (ASK)

Phase-shift keying (PSK)

Frequency-shift keying (FSK)



- Three basic signaling schemes in *M*-ary digital communications



Modulation Schemes Classification

- ***Linear modulation:*** the amplitude of the transmitted signal, *s(t),* varies linearly with the modulating digital signal, *m(t)*.
  - *Bandwidth efficient but power inefficient*
  - Example: ASK, QPSK
- ***Nonlinear modulation:*** the amplitude of the transmitted signal, *s(t),* does not vary linearly with the modulating digital signal
  - *Power efficient but bandwidth inefficient*
  - Example: FSK, constant envelope modulation

Constellation Diagram

- *A graphical representation of the complex envelope of each possible signal*
- The x-axis represents the in-phase component and the y-axis represents the quadrature component of the complex envelope
- The distance between signals on a constellation diagram relates to *how different the modulation waveforms are* and *how well a receiver can differentiate between them* when random noise is present.

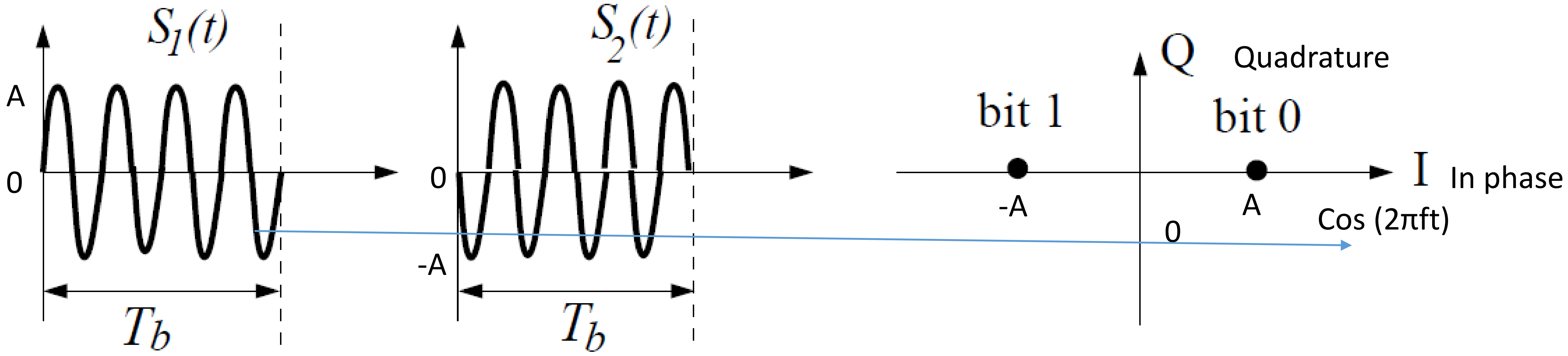
Geometric Representation (1)

- *Digital modulation involves choosing a particular analog signal waveform $s_i(t)$ from a finite set S of possible signal waveforms based on the information bits applied to the modulator.*

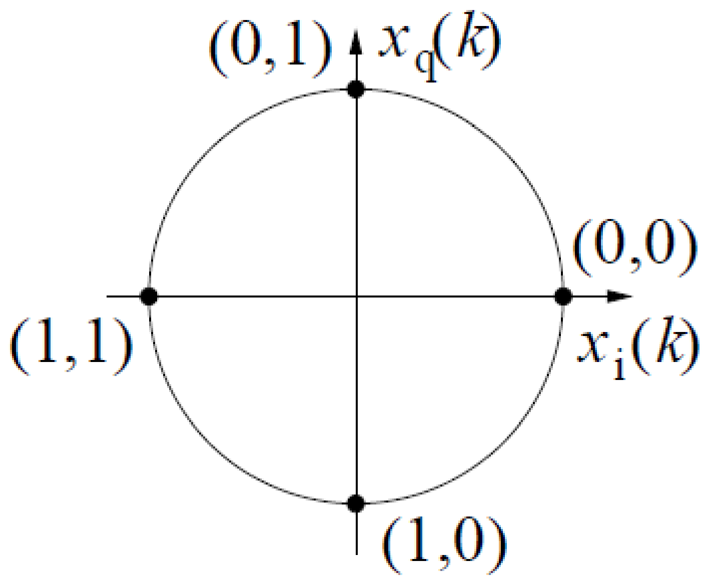
- For *binary modulation* schemes, a binary information bit is mapped directly to a signal and S contains only 2 signals, representing 0 and 1.

- For *M-ary modulations*, S contains more than 2 signals and each represents more than a single bit of information. With a signal set of size M, it is possible to transmit up to $log_2 M$ bits per signal.

- One bit per symbol, note the mapping from bits to symbols in constellation diagram, where quadrature branch is not used

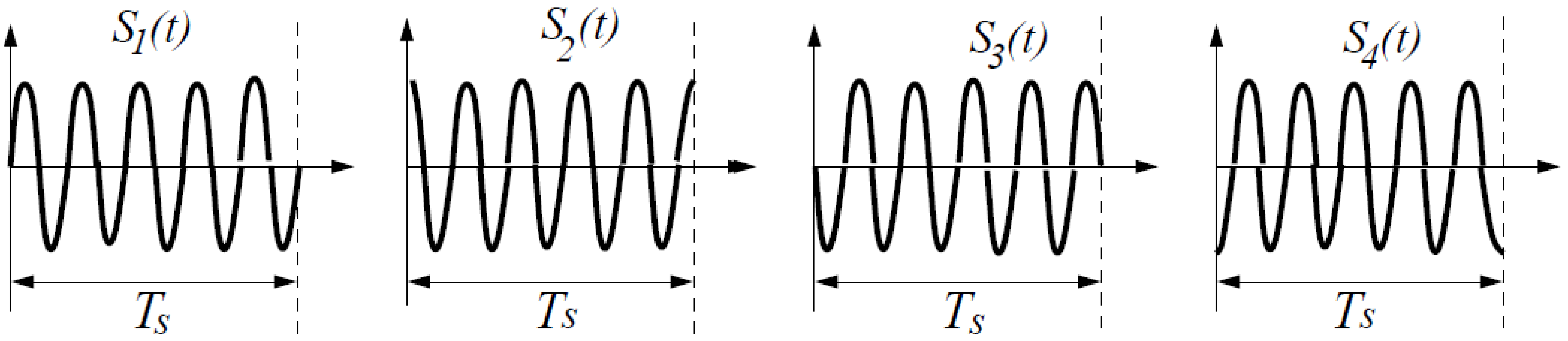


- Two bits per symbol with a minimum phase separation of $\frac{\pi}{2}$
- A QPSK constellation diagram:
  (A "different" one shown in slide **42**)



- Modulation signal set

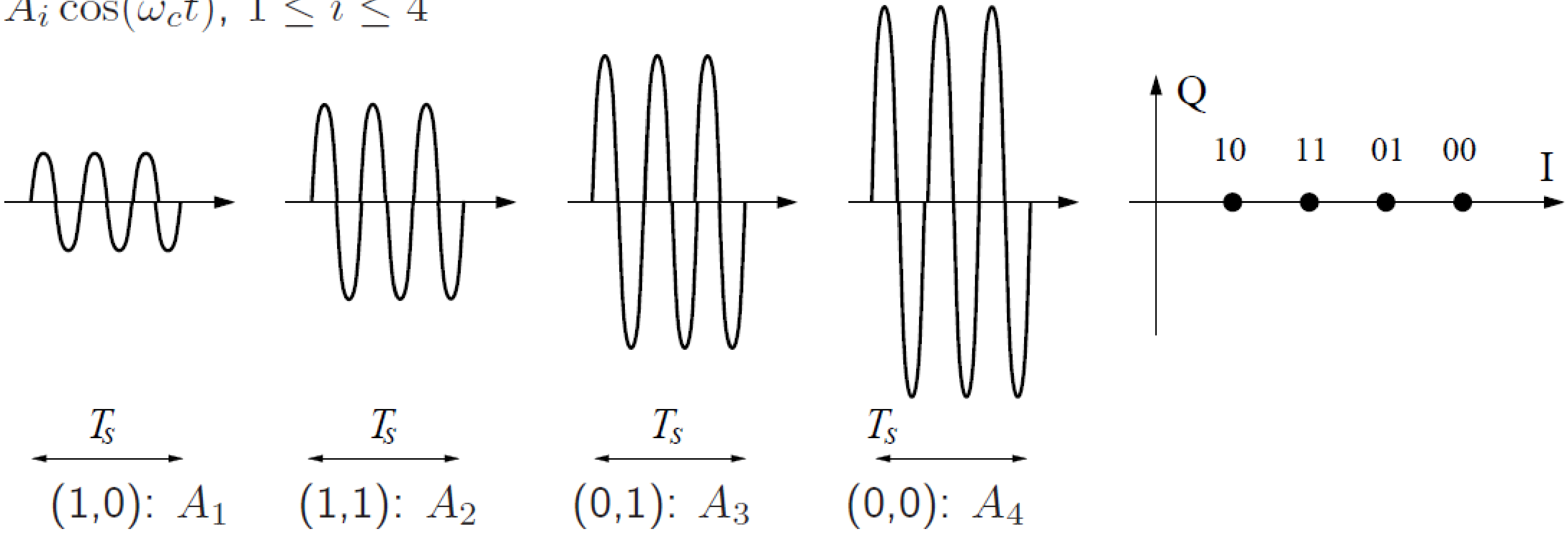
$$s_i(t) = A\cos(\omega_c t + \phi_i), \quad 1 \leq i \leq 4$$



bit (0,0): $\phi_1 = 0$ bit (0,1): $\phi_2 = \frac{\pi}{2}$ bit (1,1): $\phi_3 = \pi$ bit (1,0): $\phi_4 = \frac{3\pi}{2}$

- Pure ASK: carrier amplitude is used to carry symbol information

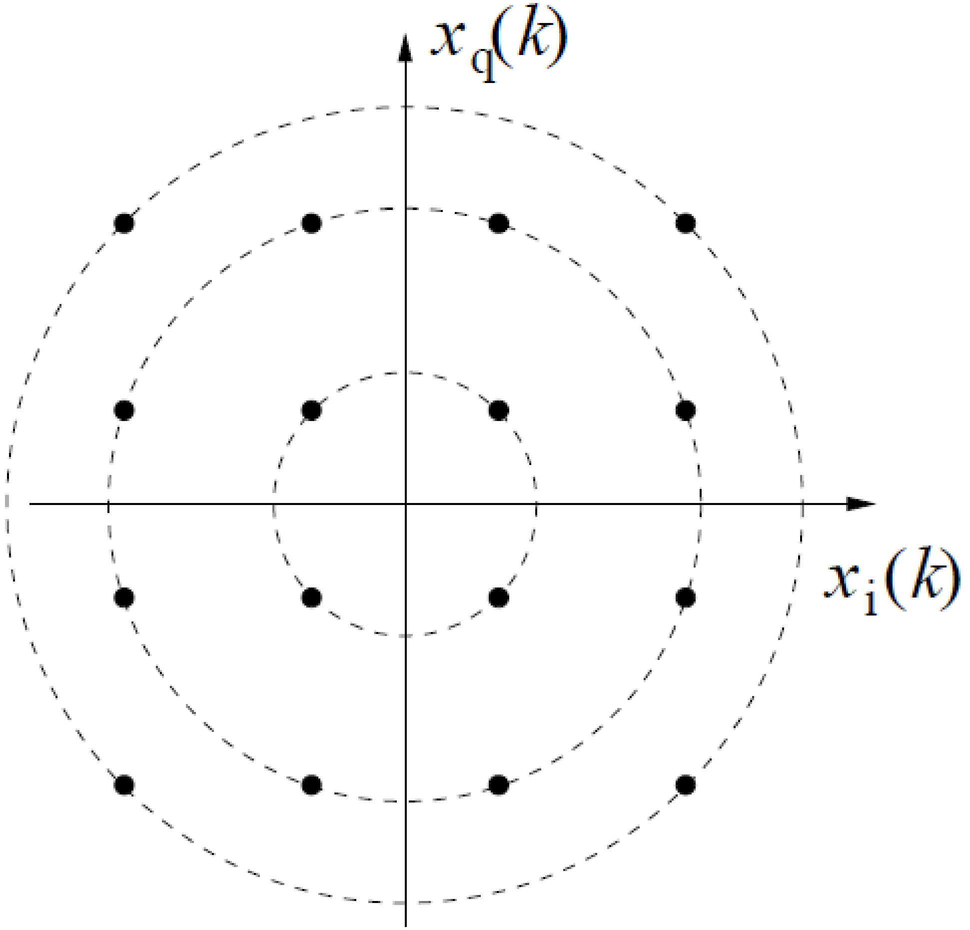
- An example of 4-ASK with constellation diagram and modulation signal set $s_i(t) = A_i \cos(\omega_c t)$, $1 \le i \le 4$



- Note quadrature branch is not used, pure ASK rarely used itself as amplitude can easily be distorted by channel

- QAM: combines features of PSK and ASK, uses both I and Q components, and is bandwidth very efficient

- An example of (squared) 16-QAM:



- Note for squared M-QAM, I and Q branches are both $\sqrt{M}$-ary (of previous slide)

- Depending on the channel quality, 64-QAM, 128-QAM, or 256-QAM are possible

Basic terms of digital communication system:

Data rate

- The measurement of the speed of a digital communication system. Its unit is usually bits/second or bit/s.

Bit Error rate

- The quality of a digital communication system is measured by bit error probability of equivalently bit error rate (BER). Its definition is

$$BER = \frac{average\ number\ of\ error\ bits}{average\ number\ of\ transmitted\ bits}$$

Bandwidth

- Bandwidth is the range of frequency spectrum used by a communication system. It is usually controlled by government and one has to pay for its usage

Power

- Power is a serious concern for any communication system. One reason for this is that any electronic device can only handle limited power level. Another issue is that the signal of one user can be the unwanted interference of another user. Minimizing the power usage of every user can lead to minimizing interference in a system. This is particularly important for a mobile cellular system.

Signal to noise ratio (S/N)

- We will see later that the relative value of the signal to noise power ration (S/N) is usually more important that the absolute value of power level.

Channel capacity – AWGN channel

$$C = B\log_2\left(1+\frac{S}{BN_0}\right)[bit/s] \text{ or}$$

$$C = B\log_2(1+\frac{S}{N})\,[bit/s] \qquad (1.1)$$

where B is the channel bandwidth, S is the average transmitted power and N_0 is the power spectral density of the additive noise. The significance of the channel capacity is as follows:

If the information rate **R** from the source is less than **C (R<C)**, than it is theoretically possible to achieve reliable (error-free) transmission through the channel by appropriate coding. On the other hand, if **R>C**, reliable transmission is not possible regardless of amount of signal processing performed at the transmitter and receiver.

Criteria of a good digital communication system:

An ideal good digital communication system should meet the following parameters:

- High data rate (high speed)
- Low error rate (high quality)
- Less bandwidth (low bandwidth cost)
- Less power (low power cost)
- Les hardware and software complexity (low equipment cost)

1.1 MATHEMATICAL MODELS FOR COMMUNICATION CHANNELS

In the design of communication systems for transmitting information through physical channels, we find it convenient to construct mathematical models that reflect the most important characteristics of the transmission medium. Then, the mathematical model for the channel is used in the design of the channel encoder and modulator at the transmitter and the demodulator and channel decoder at the receiver. Next, we provide a brief description of the channel models that are frequently used to characterize many of the physical channels that we encounter in practice.

The Additive Noise Channel

The simplest mathematical model for a communication channel is the additive noise channel, illustrated in Figure 1.2. In this model, the transmitted signal $s(t)$ is corrupted by an additive random noise process $n(t)$. Physically, the additive noise process may arise from electronic components and amplifiers at the receiver of the communication system or from interference encountered in transmission (as in the case of radio signal transmission).

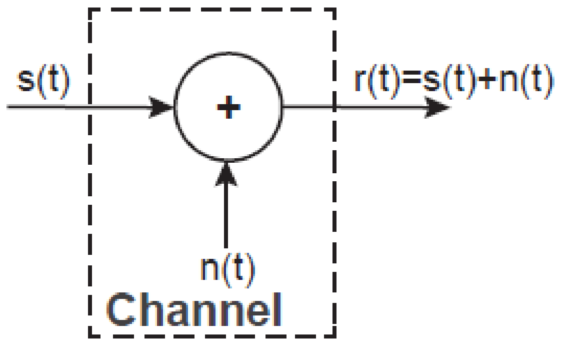


Figure 1.2 The additive noise channel

If the noise is introduced primarily by electronic components and amplifiers at the receiver, it may be characterized as thermal noise. This type of noise is characterized statistically as a Gaussian noise process. Hence, the resulting mathematical model for the channel is usually called the ***Additive Gaussian Noise Channel (AWGN)***. Because this channel model applies to a broad class of physical communication channels and because of its mathematical tractability, this is the predominant channel model used in our communication system analysis and design.

Channel attenuation is easily incorporated into the model.

$$r(t) = \alpha s(t) + n(t) \tag{1.2}$$

When the signal undergoes attenuation in transmission through the where α is the attenuation factor.

1.2 CHANNEL CAPACITY FOR DIGITAL COMMUNICATION

1.2.1 Shannon Capacity and Interpretation

The amount of noise present in the receiver can be represented in terms of its power $\boldsymbol{N} = \frac{\boldsymbol{v_n^2}}{\boldsymbol{R_{ch}}}$, where R_{ch} is the characteristic impedance of the channel, as seen by the receiver and v_n is the rms noise voltage. Similarly, the message bearing signal can be represented by its power we can represent a typical message in terms of its average signal power $\boldsymbol{S} = \frac{\boldsymbol{v_s^2}}{\boldsymbol{R_{ch}}}$, where v_s is the rms voltage of the signal. Now, it is reasonable to assume that the signal and noise are uncorrelated i.e., they are not related in any way and we cannot predict one from the other. If 'P_r' is the total power received due to the combination of signal and noise, which are uncorrelated random processes, we can write $\boldsymbol{v_r^2} = \boldsymbol{v_s^2} + \boldsymbol{v_n^2}$, i.e.,

$$P_r = S + N \tag{1.7}$$

c

Interpretation of Shannon - Hartley channel capacity $C = B\,log_2(1+\frac{S}{N})$:

a) We observe that the capacity of a channel can be increased by either

- *increasing the channel bandwidth* or
- *increasing the signal power* or
- *reducing the in-band noise power* or
- *any judicious combination of the three.*

Each approach in practice has its own merits and demerits. It is indeed, interesting to note that, all practical digital communication systems, designed so far, operate far below the capacity promised by Shannon-Hartley equation and utilizes only a fraction of the capacity. There are multiple yet interesting reasons for this. One of the overriding requirements in a practical system is sustained and reliable performance within the regulations in force. However, advances in coding theory (especially turbo coding), signal processing techniques and VLSI techniques are now making it feasible to push the operating point closer to the Shannon limit

b) If, $\boldsymbol{B} \to \infty$, we apparently have infinite capacity but it is not true. As $\boldsymbol{B} \to \infty$, the in-band noise power, N also tends to infinity [$\boldsymbol{N} = \boldsymbol{N_0 B}$, N_0: single-sided noise power spectral density, a constant for AWGN] and hence, $\boldsymbol{\frac{S}{N}} \to \mathbf{0}$ for any finite signal power 'S' and $\mathbf{log_2(1+\frac{S}{N})}$ also tends to zero. So, it needs some more careful interpretation and we can expect an asymptotic limit.

At capacity, the bit rate of transmission R_b = C and the duration of a bit $= \frac{1}{R_b} = \frac{1}{C}$. If the energy received per information bit is E_b, the signal power S can be expressed as, S = energy received per unit time = $E_bR_b = E_bC$. So, the signal-to-noise ratio $\frac{S}{N}$ can be expressed as,

$$\frac{S}{N} = \frac{E_bC}{N_0B} \tag{1.8}$$

Now, we can write

$$\frac{C}{B} = \log_2(1 + \frac{E_bC}{N_0B}) \tag{1.9}$$

This implies

$$\frac{E_b}{N_0} = \frac{B}{C}\left(2^{\frac{C}{B}} - 1\right) \tag{1.10}$$

$$\cong \frac{B}{C}\left[\left(1 + \frac{C}{B} ln2\right) - 1\right], for\ B \gg C \tag{1.11}$$

$$= \log_e 2\,, for\ B \gg C \tag{1.12}$$

$$= -1{,}6\ dB \tag{1.13}$$

So, the limiting $\frac{E_b}{N_0}$, in dB is **-1.6 dB**. So, ideally, a system designer can expect to achieve almost errorless transmission only when the $\frac{E_b}{N_0}$ is more than -1.6 dB and there is no constraint in bandwidth.

c) In the above observation, we set $\mathbf{R_b} = \mathbf{C}$ to appreciate the limit $\frac{E_b}{N_0}$ and we also saw that if $\mathbf{R_b} > \mathbf{C}$, the noise v_n is capable of distorting the group of 'b' information bits. We say that the bit rate has exceeded the capacity of the channel and hence errors are not controllable by any means.

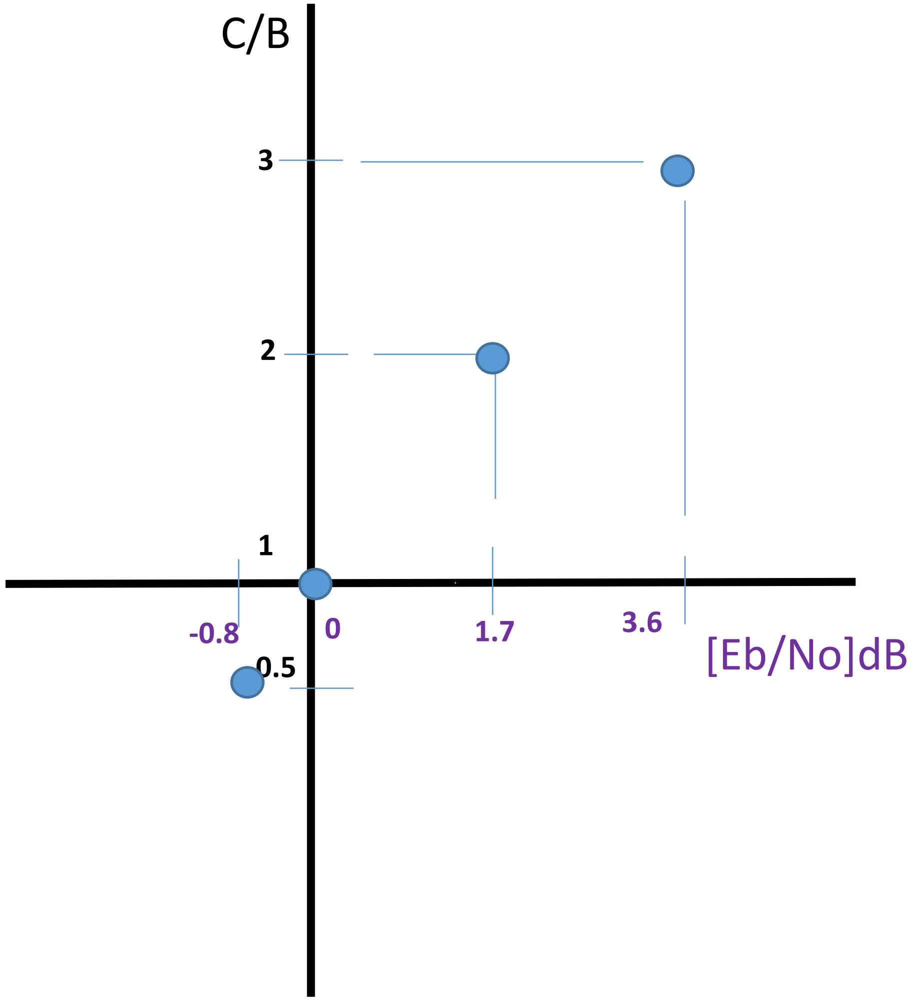
$$C = B\,log_2\left(1+\frac{CE_b}{BN_0}\right) \longrightarrow \frac{E_b}{N_0} = \frac{B}{C}\left(2^{\frac{C}{B}}-1\right)$$

a) C/B=1; 1(2-1) = Eb/No; 10log(Eb/No) = 10 log1 = 0 [dB]

b) C/B=2; 1/2(4-1) = Eb/No; 10log(Eb/No) = 10 log(3/2) = 1,7 [dB]

c) C/B=3; 1/3(8-1) = Eb/No; 10log(Eb/No) = 10 log(7/3) = 3,6 [dB]

d) C/B=1/2; $2(2^{\frac{1}{2}}-1)$ = Eb/No; 10log(Eb/No) = 10 log(0.82) = -0,817 [dB]



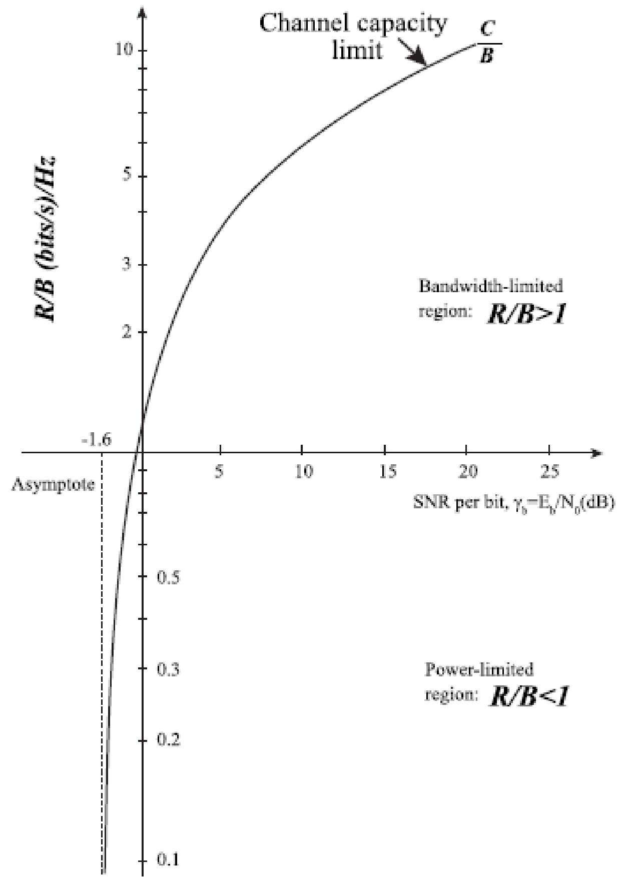
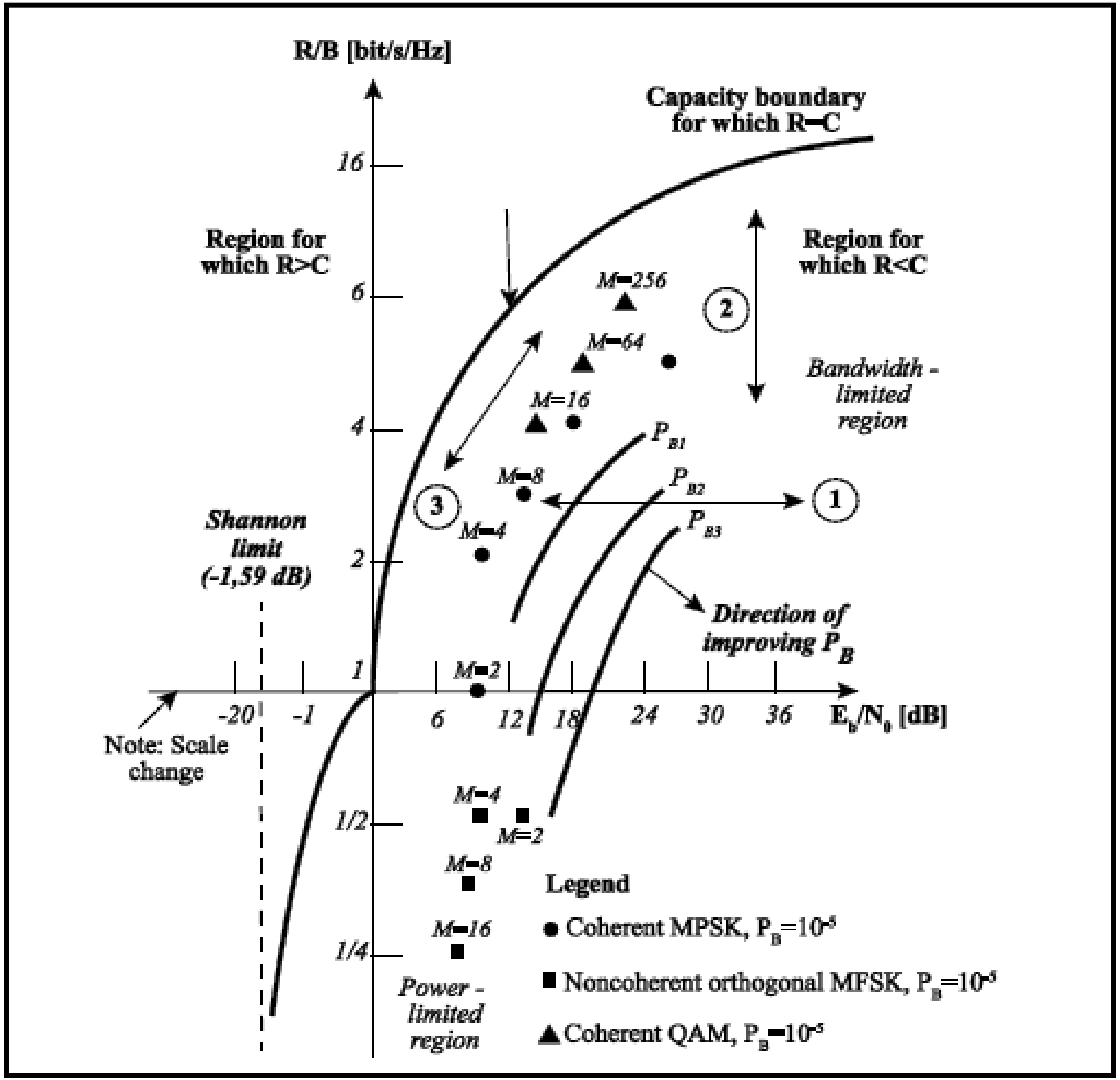


Figure 1.5 Interpretation of Shannon-Hartley channel capacity

To reiterate, all practical systems obey the inequality $R_b < C$ and most of the civilian digital transmission systems utilize the available bandwidth efficiently, which means B (in Hz) and C (in bits per second) are comparable. For bandwidth efficient transmission, the strategy is to increase the bandwidth factor $\frac{R_b}{B}$ while $R_b < C$. This is achieved by adopting suitable modulation and reception strategies.


R/B [bit/s/Hz]
Capacity boundary for which R=C
Region for which R>C
Region for which R<C
Bandwidth - limited region
Shannon limit (-1,59 dB)
Direction of improving P_B
Note: Scale change
Power - limited region
E_b/N_0 [dB]
16
6
4
2
1
1/2
1/4
-20
-1
6
12
18
24
30
36
M=256
M=64
M=16
M=8
M=4
M=2
M=4
M=2
M=8
M=16
P_B1
P_B2
P_B3
1
2
3
Legend
Coherent MPSK, P_B=10^-5
Noncoherent orthogonal MFSK, P_B=10^-5
Coherent QAM, P_B=10^-5

1.2.2 Hartley Channel Capacity

Consider a noise free channel where the limitation on data rate is simply the bandwidth of the signal. Nyquist states that if the rate of signal transmission is 2B, then a signal with frequencies no greater than B is sufficient to carry the signal rate. Conversely given a bandwidth of B, the highest signal rate that can be carried is 2B. This limitation is due to the effect of inter symbol interference, such as is produced by delay distortion.

If the signals to be transmitted are binary (two voltage levels), then the data rate that can be supported by B Hz is 2B [bps]. However signals with more than two levels can be used; that is, each signal element can represent more than one bit. For example, if four possible voltage levels are used as signals, then each signal element can represent two bits. With multilevel signaling, the Nyquist formulation becomes:

$$C = 2B \log_2 M \quad (1.14)$$

where M is the number of discrete signal or voltage levels.

NOISE IN DIGITAL COMMUNICATION SYSTEM

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{n}{\sigma}\right)^2\right]$$

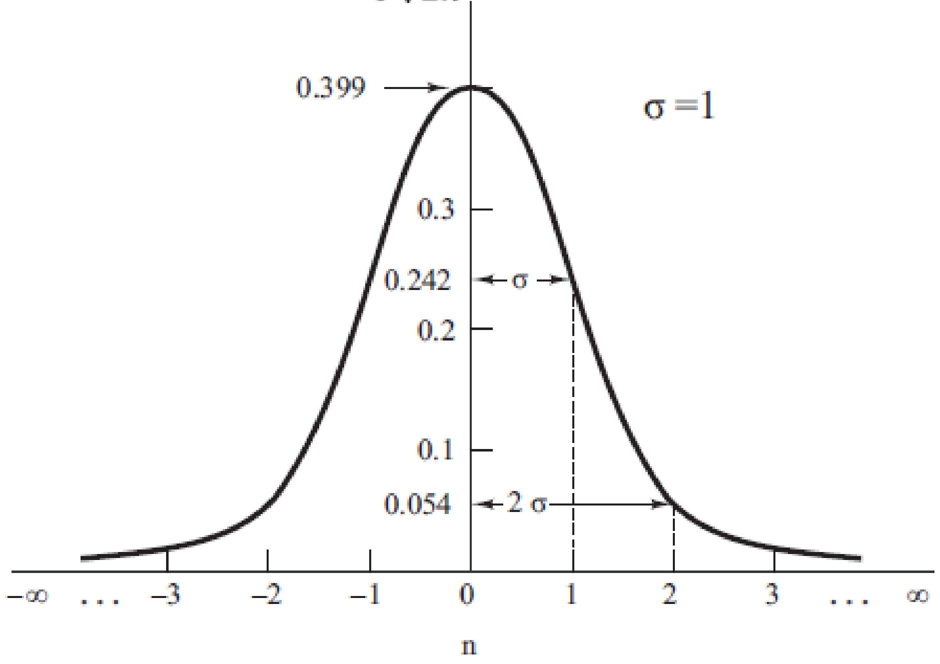
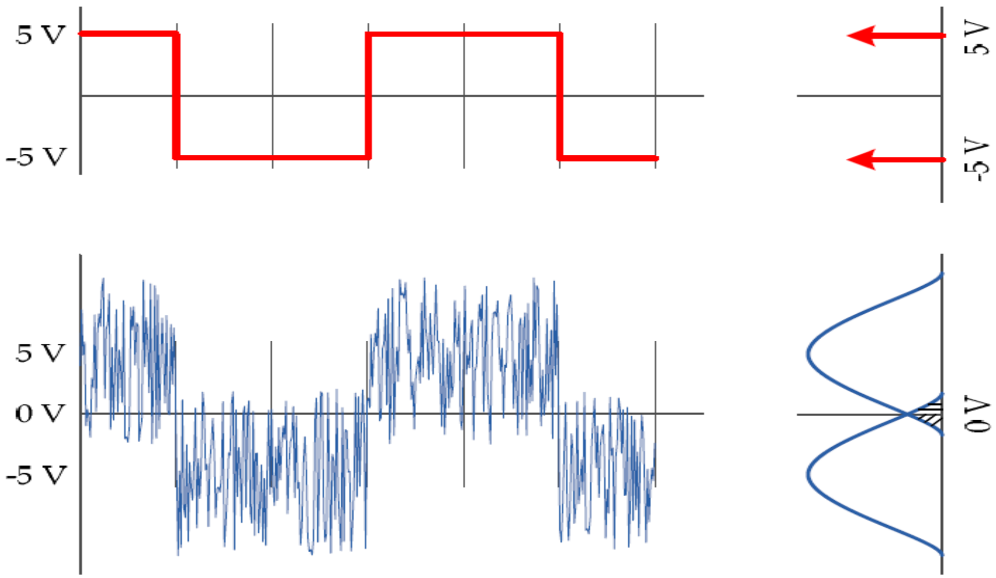


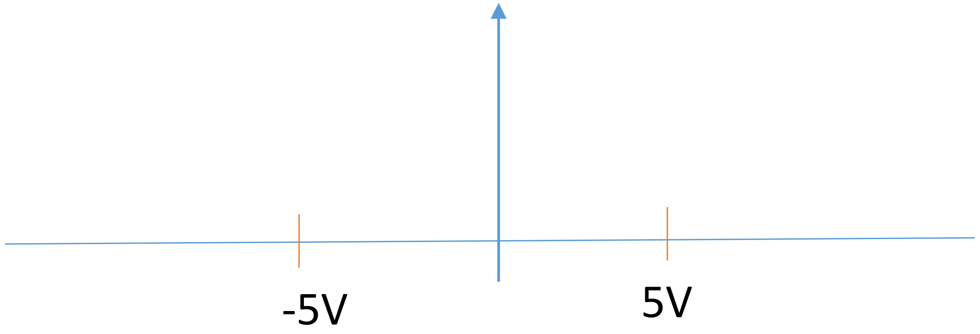
Figure 1.8 Normalized ($\sigma = 1$) Gaussian probability density function

a) No noise



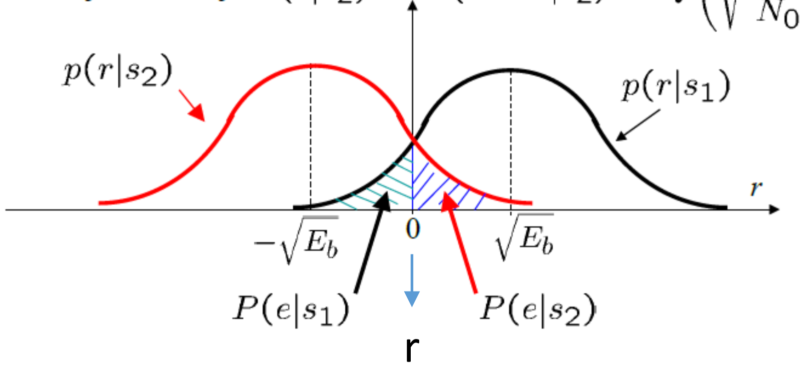
b) With noise

a) No noise



b) With noise

Due to symmetry $P(e|s_2) = P(r > 0|s_2) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$



$$r = \frac{N_0}{4\sqrt{E_b}} \log \frac{1-p}{p}$$

1.3.1 White Noise

The primary spectral characteristic of thermal noise is that its power spectral density is the same for all frequencies of interest in most communication systems; in other words, a thermal noise source emanates an equal amount of noise power per unit bandwidth at all frequencies—from dc to about 10^{12} Hz. Therefore, a simple model for thermal noise assumes that its power spectral density $\Phi_n(f)$ is flat for all frequencies, as shown in Figure 1.9 a), and is denoted as

$$\Phi_n(f) = \frac{N_0}{2}\left[\frac{W}{Hz}\right], -\infty \leq f \leq \infty \tag{1.18}$$

where the factor of 2 is included to indicate that $\Phi_n(f)$ is a *two-sided power spectral density*. When the noise power has such a uniform spectral density we refer to it as *white noise*. The adjective "white" is used in the same sense as it is with white light, which contains equal amounts of all frequencies within the visible band of electromagnetic radiation. The autocorrelation function of white noise is given by the inverse Fourier transform of the noise power spectral density, denoted as follows:

$$\phi_n(\tau) = \mathcal{F}^{-1}\{\Phi_n(f)\} = \int_{-\infty}^{\infty} \Phi_n(\tau) e^{j2\pi f\tau} df = \frac{N_0}{2}\delta(\tau) \tag{1.19}$$

Thus the autocorrelation of white noise is a delta function weighted by the factor $\frac{N_0}{2}$ and occurring at $\tau = 0$, as seen in Figure 1.9 b). Note that $\phi_n(\tau)$ is zero for $\tau \neq 0$; that is, any two different samples of white noise, no matter how close together in time they are taken, are uncorrelated.

The average power P_n of white noise is infinite because its bandwidth is infinite. This can be seen to yield:

$$P_n = \int_{-\infty}^{\infty} \frac{N_0}{2} df = \infty \tag{1.20}$$

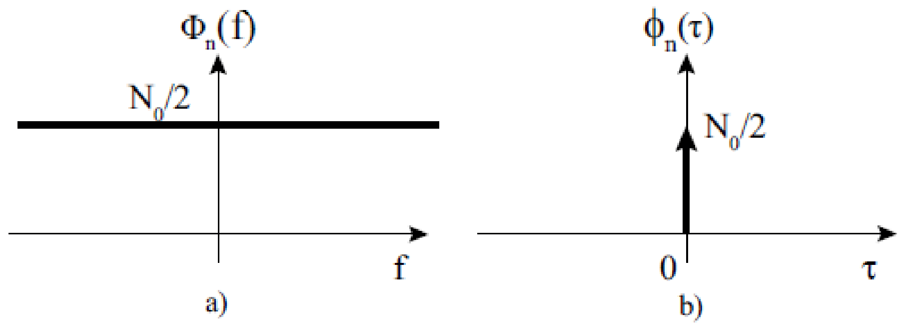


Figure 1.9 a) Power spectral density of white noise, b) autocorrelation function of white noise

Although white noise is a useful abstraction, no noise process can truly be white; however, the noise encountered in many real systems can be assumed to be approximately white. We can only observe such noise after it has passed through a real system which will have a finite bandwidth. Thus, as long as the bandwidth of the noise is appreciably larger than that of the system, the noise can be considered to have an infinite bandwidth.

The delta function in Equation 1.19 means that the noise signal *n(t)* is totally decorrelated from its time-shifted version, for any $\tau > 0$. Equation 1.19 indicates that any two different samples of a white noise process are uncorrelated. Since thermal noise is a Gaussian process and the samples are uncorrelated, the noise samples are also independent. Therefore, the effect on the detection process of a channel with **additive white Gaussian noise (AWGN)** is that the noise affects each transmitted symbol independently. Such a channel is called a memoryless channel. The term "additive" means that

1.3.2 *Thermal Noise*

Thermal noise is caused by the thermal motion of electrons in all conductors. It is generated in the lossy coupling between an antenna and receiver and in the first stages of the receivers. The noise power spectral density is constant at all frequencies up to about 10^{12} Hz, giving rise to the name white noise. The thermal noise process in communication receivers is modeled as an additive white Gaussian noise (AWGN) process. The physical model for thermal or Johnson noise is a noise generator with an open-circuit mean square voltage of $4kT°BR$, where:

- κ = Botlzmann's constant = 1.38×10^{-23} [J/K] or [W/K-Hz] = -228,6 [dBW/K-Hz]
- $T°$ = temperature [K]
- B = bandwidth [Hz]
- R = resistance [Ω]

$T_{(K)} = T_{(°C)} + 273{,}15$

$T_{(K)} = 10\ °C + 273{,}15 = 283{,}15\ K$
$T_{(K)} = 20\ °C + 273{,}15 = 293{,}15\ K$
$0_{(K)} = -273{,}15\ °C$

The maximum thermal noise power N that could be coupled from the noise generator into the front end of an amplifier is:

$$N = \kappa T° B\ [W] \tag{1.21}$$

Thus, the maximum single-sided noise power spectral density N_0 (noise power in a 1 Hz bandwidth) available at the amplifier input is

$$N_0 = \frac{N}{B} = \kappa T°\ [W/Hz] \tag{1.22}$$

Problem 1

Using a noise generator with mean-square voltage equal to 4κT°BR, demonstrate that the maximum amount of noise power that can be coupled from this source into amplifier is $N_i = \kappa T° BW$

Solution

A theorem from network theory states that maximum power is delivered to a load when the value of the load impedance is made equal to the complex conjugate of the generator impedance. In this case the generator impedance is a pure resistance *R;* therefore, the condition for maximum power transfer is fulfilled when the input resistance of the amplifier equals *R*.

Figure 1.11 illustrates such a network. The input thermal noise source is represented by an electrically equivalent model consisting of a noiseless source resistor in series with an ideal voltage generator whore rms noise voltage is $\sqrt{4\kappa T° BR}$.

The input resistance of the amplifier is made equal to *R*. the noise voltage delivered to the amplifier input is just one – half the generator voltage, following basic circuit principles. The noise power delivered to the amplifier input can accordingly be expressed as:

$$N_i = \frac{(\sqrt{4\kappa T° BR}/2)^2}{R} = \frac{4\kappa T° BR}{4R} = \kappa T° B$$

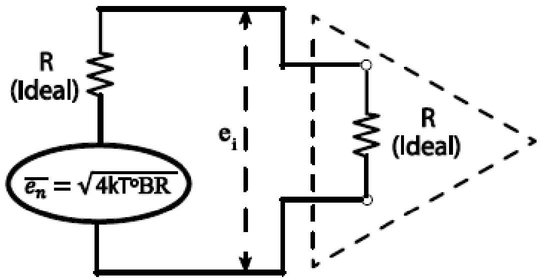


Figure 1.11 Electrical model of maximum available thermal noise power at amplifier input

dB in Communications

- The db (decibel) is a relative unit of measurement commonly used in communications for providing a reference for input and output levels.
  - Power gain or loss.
- Decibels are used to specify measured and calculated values in
  - audio systems, microwave system gain calculations, satellite system link-budget analysis, antenna power gain, light-budget calculations and in many other communication system measurements
  - In each case the dB value is calculated with respect to a standard or specified reference.

Calculation of dB

- The dB value is calculated by taking the log of the ratio of the measured or calculated power (P2) with respect to a reference power (P1).

- The result is multiplied by 10 to obtain the value in dB.

$$\text{dB} = 10\log_{10}\frac{P_2}{P_1}$$

- It can be modified to provide a dB value based on the ratio of two voltages. By using the power relationship P = V^2/R

$$\text{dB} = 10\log_{10}\frac{P_2}{P_1} = 10\log_{10}\frac{V_2^2/R}{V_1^2/R} = 20\log_{10}\frac{V_2}{V_1}$$

Definitions of dBm and dBW

- dBm indicates that the specified dB level is relative to a 1 milliwatt reference.

$$\mathrm{dBm} = 10\log_{10}\frac{P_2}{0.001\mathrm{W}}$$

- If Power is expressed in watts instead of milliwatts.
  - the dB unit is obtained with respect to 1 watt and the dB values are expressed as dBW.

$$\mathrm{dBW} = 10\log_{10}\frac{P_2}{1\ \mathrm{W}}$$

Signal $s_m(t)$, $1 \le m \le M$, $t \in [0, T_s)$

- Signaling interval: T_s (For convenience, we will use T instead sometimes.)
- Signaling rate (or symbol rate): $R_s = \frac{1}{T_s}$
- (Equivalent) Bit interval: $T_b = \frac{T_s}{\log_2 M}$
- (Eqiuvalent) Bit rate: $R_b = \frac{1}{T_b} = R_s \log_2 M$
- Average signal energy (assume equal-probable in message m)

$$\mathcal{E}_{\text{avg}} = \frac{1}{M} \sum_{m=1}^{M} \int_0^{T_s} |s_m(t)|^2 \, dt$$

- (Equivalent) Average bit energy: $\mathcal{E}_{\text{bavg}} = \frac{\mathcal{E}_{\text{avg}}}{\log_2 M}$
- Average power: $P_{\text{avg}} = \frac{\mathcal{E}_{\text{avg}}}{T_s} = R_s \mathcal{E}_{\text{avg}} = \frac{\mathcal{E}_{\text{bavg}}}{T_b} = R_b \mathcal{E}_{\text{bavg}}$

Efektivna hodnota (rms), Maximalna hodnota, Vykon P, Energia E

(rms = root mean square = efektivna hodnota)

$$X_{max} \equiv X \qquad I_{rms} = \frac{I_{max}}{\sqrt{2}} \qquad U_{rms} = \frac{U_{max}}{\sqrt{2}}$$

$$\boldsymbol{P = \frac{U_{rms}^2}{R} = \frac{\frac{U_{max}^2}{\sqrt{2}}}{R} = \frac{U_{max}^2}{2R} = \frac{A_{max}^2}{2R}}$$

R – odpor [ohm]

$$\boldsymbol{A_{max} \equiv A = \sqrt{2RP} = \sqrt{2R\frac{E}{T}}\ ;}$$

$$P = \frac{E}{T}\ ;$$

$$E = \frac{A^2 T}{2R}$$