Problem 11.12 Compute the noise spectral density in watts per hertz of:
(a) an ideal resistor at nominal temperature of 290°K;
(b) an amplifier with an equivalent noise temperature of 22,000°K.

Solution

(a) From Eq. (11.19), the noise power spectral density is

$$\begin{aligned} N_0 &= kT_e \\ &= 1.38\times10^{-23}\times290 \\ &= 4.0\times10^{-21}\ \text{W/Hz} \end{aligned}$$

(b) From Eq. (11.19), the noise power spectral density is

$$\begin{aligned} N_0 &= kT_e \\ &= 1.38\times10^{-23}\times22000 \\ &= 3.04\times10^{-19}\ \text{W/Hz} \end{aligned}$$

Example:

What is the noise level, in dBm, of a resistor at 17°C (room temperature) over a 1 MHz bandwidth?

N = kTB = $(1.38 \times 10^{-23}) \times (273 + 17) \times (1 * 10^{6})$ = $1.37 \times 10^{-17} \times 290$ = 4.0×10^{-15} [Joules/Second] = 4×10^{-15} [Watts]

In dBm:

1×10^{-15} Watts = 1×10^{-12} mW = -120 [dBm] lebo $10 \log 10^{-12}$ = -120 [dBm]

Problem 11.13 For the two cases of Problem 11.12, compute the pre-detection SNR when the received signal power is:
(a) -60 dBm and the receive bandwidth is 1 MHz;
(b) -90 dBm and the receive bandwidth is 30 kHz.
Express the answers in both absolute terms and decibels.

Solution
(a) The signal power is obtained by converting -60 dBm to watts

$$S = 10^{(-60/10)} = 10^{-6}\,\text{mW} = 10^{-9}\,\text{W}$$

The noise power from the ideal resistor is from Eq. (11.13)

$$N = kT_eB_N \qquad = 1{,}38 \times 10^{-23} \times 290 \times 10^{6}$$
$$= 4.0\times10^{-21}\times(10^{6})$$
$$= 4.0\times10^{-15}\,\text{W}$$

The SNR is the ratio of the two

$$SNR = \frac{S}{N} = \frac{10^{-9}}{4.0\times10^{-15}} = 2.5\times10^{5} \sim 54\text{ dB}$$

A similar calculation for the amplifier of the previous problem results in

$$SNR = \frac{S}{N} = \frac{10^{-9}}{3.04\times10^{-19}\times10^{6}} = 2.94\times10^{3} \sim 34.7\text{ dB}$$

(b) The signal power is obtained by converting -90 dBm to watts

$$S = 10^{(-90/10)} = 10^{-9}\,\text{mW} = 10^{-12}\,\text{W}$$

The noise power from the ideal resistor is from Eq. (11.13)

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$dB_m \sim 1\,mW$

$[S]_{dBm} = 10\log S\ [mW]$

$-60\,dBm = 10\log S$

$S = 10^{-60/10} = 10^{-6}\,[mW] = 10^{-9}\,[W]$

Problem 11.13 continued

$$N = kT_eB_N$$
$$= 4.0\times10^{-21}\times(30\times10^{3})$$
$$= 1.2\times10^{-16}\,\text{W}$$

The SNR is the ratio of the two

$$SNR = \frac{S}{N} = \frac{10^{-12}}{1.2\times10^{-16}} = 8.3\times10^{3} \sim 39.2\text{ dB}$$

A similar calculation for the amplifier of the previous problem results in

$$SNR = \frac{S}{N} = \frac{10^{-12}}{3.04\times10^{-19}\times(30\times10^{3})} = 1.1\times10^{2} \sim 20.4\text{ dB}$$

Problem 11.14 A wireless local area network transmits a signal that has a noise bandwidth of approximately 6 MHz. If the signal strength at the receiver input terminals is –90 dBm and the receiver noise figure is 8 dB, what is the pre-detection signal-to-noise ratio?

Solution

The signal power is obtained by converting -90 dBm to watts

$$S = 10^{(-90/10)} = 10^{-9}\,\text{mW} = 10^{-12}\,\text{W}$$

The noise power with an 8 dB noise figure F is from Eqs. (11.15) and (11.16)

$$\begin{aligned} N &= kT_0FB \\ &= 1.38\times10^{-23}\times(290)\times10^{8/10}\times(6\times10^{6}) \\ &= 1.52\times10^{-13}\,\text{W} \end{aligned}$$

The pre-detection SNR is the ratio of the two

$$SNR = \frac{S}{N} = \frac{10^{-12}}{1.52\times10^{-13}} = 6.6 \sim 8.2\ \text{dB}$$

F - indicates how much noise the receiver electronics add to the thermal noise.

$F\,[dB] = 10 \log F$

$8\,[dB] = 10 \log F$

$F = 10^{8/10}$

Problem 11.20 If a receiver has a sensitivity of –90 dBm and a 12 dB noise figure what is minimum pre-detection signal-to-noise ratio of an 8 kHz signal?

<u>Solution</u>

The noise in an 8 kHz bandwidth for a receiver with an [redacted] 12 dB noise figure is, from Eqs. (11.15) and (11.16),

$$
\begin{aligned}
N &= kT_0FB \\
&= 1.38\times10^{-23}\times(290)\times\left(10^{12/10}\right)\times(8\times10^{3}) \\
&= 5.07\times10^{-16}\ \mathrm{W}
\end{aligned}
$$

The receiver sensitivity is defined as the minimum received signal power that will provide a demodulated signal with acceptable performance, thus the minimum signal power is S = -90 dBm ~ 10^{-12} W. The minimum pre-detection SNR is the ratio of the two

$$SNR = \frac{S}{N} = \frac{10^{-12}}{5.07\times10^{-16}} = 1.97\times10^{3} \sim 32.9\ \mathrm{dB}$$

Problem 10.28. A binary FSK system transmits data at the rate of 2.5 megabits per second. During the course of transmission, white Gaussian noise of zero mean and power spectral density 10^{-20} watts per hertz is added to the signal. In the absence of noise, the amplitude of the received signal is 1 μV across 50 ohm impedance. Determine the average probability of error assuming coherent detection of the binary FSK signal.

Solution

The average probability of error for coherent FSK is

$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

from Eq. (10.68). For this example, we have noise power spectral density is

$$N_0 = 2\times 10^{-20} \text{ watts / Hz}$$

and the energy per bit is

$$E_b = \frac{1}{2}\frac{A_c^2 T}{R},$$

In the text, we have nominally assumed the resistance is 1 ohm and omitted it. In this problem we use the resistance of R = 50 ohms. The symbol duration is $T = \dfrac{1}{2.5\times 10^6}$ seconds and the amplitude of received signal is A_c = 1μV. Therefore,

$$E_b = \frac{1}{2}\times\frac{1\times 10^{-12}}{50}\times\frac{1}{2.5\times 10^6}$$
$$= 4\times 10^{-21} \text{ watts / Hz}$$

Substituting the above values into the expression for P_e and we have the probability of error is

$$P_e = Q(\sqrt{0.2}) = Q(0,447) \doteq 0,326$$

[redacted]

$$A = \sqrt{2}A_{rms}$$

$$P = \frac{A_{rms}^2}{R} = \frac{A^2}{2R}$$

$$A = \sqrt{2PR}$$

$$A = \sqrt{\frac{2RE}{T}}$$

$$E = \frac{A^2 T}{2R}$$

Table 1: Values of $Q(x)$ for $0 \le x \le 9$

x	$Q(x)$	x	$Q(x)$	x	$Q(x)$	x	$Q(x)$
0.00	0.5	2.30	0.010724	4.55	2.6823×10^{-6}	6.80	5.231×10^{-12}
0.05	0.48006	2.35	0.0093867	4.60	2.1125×10^{-6}	6.85	3.6925×10^{-12}
0.10	0.46017	2.40	0.0081975	4.65	1.6597×10^{-6}	6.90	2.6001×10^{-12}
0.15	0.44038	2.45	0.0071428	4.70	1.3008×10^{-6}	6.95	1.8264×10^{-12}
0.20	0.42074	2.50	0.0062097	4.75	1.0171×10^{-6}	7.00	1.2798×10^{-12}
0.25	0.40129	2.55	0.0053861	4.80	7.9333×10^{-7}	7.05	8.9459×10^{-13}
0.30	0.38209	2.60	0.0046612	4.85	6.1731×10^{-7}	7.10	6.2378×10^{-13}
0.35	0.36317	2.65	0.0040246	4.90	4.7918×10^{-7}	7.15	4.3389×10^{-13}
0.40	0.34458	2.70	0.003467	4.95	3.7107×10^{-7}	7.20	3.0106×10^{-13}
0.45	0.32636	2.75	0.0029798	5.00	2.8665×10^{-7}	7.25	2.0839×10^{-13}
0.50	0.30854	2.80	0.0025551	5.05	2.2091×10^{-7}	7.30	1.4388×10^{-13}
0.55	0.29116	2.85	0.002186	5.10	1.6983×10^{-7}	7.35	9.9103×10^{-14}
0.60	0.27425	2.90	0.0018658	5.15	1.3024×10^{-7}	7.40	6.8092×10^{-14}
0.65	0.25785	2.95	0.0015889	5.20	9.9644×10^{-8}	7.45	4.667×10^{-14}
0.70	0.24196	3.00	0.0013499	5.25	7.605×10^{-8}	7.50	3.1909×10^{-14}
0.75	0.22663	3.05	0.0011442	5.30	5.7901×10^{-8}	7.55	2.1763×10^{-14}
0.80	0.21186	3.10	0.0009676	5.35	4.3977×10^{-8}	7.60	1.4807×10^{-14}
0.85	0.19766	3.15	0.00081635	5.40	3.332×10^{-8}	7.65	1.0049×10^{-14}
0.90	0.18406	3.20	0.00068714	5.45	2.5185×10^{-8}	7.70	6.8033×10^{-15}
0.95	0.17106	3.25	0.00057703	5.50	1.899×10^{-8}	7.75	4.5946×10^{-15}
1.00	0.15866	3.30	0.00048342	5.55	1.4283×10^{-8}	7.80	3.0954×10^{-15}
1.05	0.14686	3.35	0.00040406	5.60	1.0718×10^{-8}	7.85	2.0802×10^{-15}
1.10	0.13567	3.40	0.00033693	5.65	8.0224×10^{-9}	7.90	1.3945×10^{-15}
1.15	0.12507	3.45	0.00028029	5.70	5.9904×10^{-9}	7.95	9.3256×10^{-16}
1.20	0.11507	3.50	0.00023263	5.75	4.4622×10^{-9}	8.00	6.221×10^{-16}
1.25	0.10565	3.55	0.00019262	5.80	3.3157×10^{-9}	8.05	4.1397×10^{-16}
1.30	0.0968	3.60	0.00015911	5.85	2.4579×10^{-9}	8.10	2.748×10^{-16}
1.35	0.088508	3.65	0.00013112	5.90	1.8175×10^{-9}	8.15	1.8196×10^{-16}
1.40	0.080757	3.70	0.0001078	5.95	1.3407×10^{-9}	8.20	1.2019×10^{-16}
1.45	0.073529	3.75	8.8417×10^{-5}	6.00	9.8659×10^{-10}	8.25	7.9197×10^{-17}
1.50	0.066807	3.80	7.2348×10^{-5}	6.05	7.2423×10^{-10}	8.30	5.2056×10^{-17}
1.55	0.060571	3.85	5.9059×10^{-5}	6.10	5.3034×10^{-10}	8.35	3.4131×10^{-17}
1.60	0.054799	3.90	4.8096×10^{-5}	6.15	3.8741×10^{-10}	8.40	2.2324×10^{-17}
1.65	0.049471	3.95	3.9076×10^{-5}	6.20	2.8232×10^{-10}	8.45	1.4565×10^{-17}
1.70	0.044565	4.00	3.1671×10^{-5}	6.25	2.0523×10^{-10}	8.50	9.4795×10^{-18}
1.75	0.040059	4.05	2.5609×10^{-5}	6.30	1.4882×10^{-10}	8.55	6.1544×10^{-18}
1.80	0.03593	4.10	2.0658×10^{-5}	6.35	1.0766×10^{-10}	8.60	3.9858×10^{-18}
1.85	0.032157	4.15	1.6624×10^{-5}	6.40	7.7688×10^{-11}	8.65	2.575×10^{-18}
1.90	0.028717	4.20	1.3346×10^{-5}	6.45	5.5925×10^{-11}	8.70	1.6594×10^{-18}
1.95	0.025588	4.25	1.0689×10^{-5}	6.50	4.016×10^{-11}	8.75	1.0668×10^{-18}
2.00	0.02275	4.30	8.5399×10^{-6}	6.55	2.8769×10^{-11}	8.80	6.8408×10^{-19}
2.05	0.020182	4.35	6.8069×10^{-6}	6.60	2.0558×10^{-11}	8.85	4.376×10^{-19}
2.10	0.017864	4.40	5.4125×10^{-6}	6.65	1.4655×10^{-11}	8.90	2.7923×10^{-19}
2.15	0.015778	4.45	4.2935×10^{-6}	6.70	1.0421×10^{-11}	8.95	1.7774×10^{-19}
2.20	0.013903	4.50	3.3977×10^{-6}	6.75	7.3923×10^{-12}	9.00	1.1286×10^{-19}
2.25	0.012224						