Deterministic signal:

No uncertainty with respect to the signal value at any time.

$$x(t) = A \cos(2\pi f t)$$

Random signal:

Some degree of uncertainty in signal values before it actually occurs.

- *thermal noise in electronic circuits due to the random movement of electrons,*
- *reflection of radio waves from different layers of ionosphere*

Real and Complex Signals

1. In both real and complex signals, the independent variable is real-valued.

2. A real signal at any given time takes its value in the set of real numbers, and a complex signal takes its value in the set of complex numbers.

3. A complex signal can in turn be represented by two real signals, such as the real-In phase and imaginary Q-quadrature parts or equivalently magnitude (amplitude) and phase values.

4. **Signals that we observe physically (using voltmeters, ammeters, oscilloscopes, etc.) are all real signals, as complex signals have no physical meaning!!!**

5. Certain mathematical models and calculations can be greatly simplified if we use complex notation.

6. In communications, a complex signal is often used to convey information about the magnitude and phase of a signal in the frequency domain.

2.4 ENERGY AND POWER SIGNALS

An electrical signal can be represented as a voltage *v(t)* or a current *i(t)* with instantaneous power *p(t)* across a resistor *R* defined by

$$p(t) = \frac{v^2(t)}{R} \tag{2.2}$$

Or

$$p(t) = i^2(t)R \tag{2.3}$$

In communication systems, power is often normalized by assuming *R* to be 1Ω, although *R* may be another value in the actual circuit. If the actual value of the power is needed, it is obtained by "denormalization" of the normalized value. For the normalized case, Equations 2.2 and 2.3 have the same form. Therefore, regardless of whether the signal is a voltage or current waveform, the normalization convention allows us to express the instantaneous power as

$$p(t) = x^2(t) \tag{2.4}$$

where *x(t)* is either a voltage or a current signal. The energy dissipated during the time interval (−T/2, T/2) by a real signal with instantaneous power expressed by Equation 2.4 can then be written as

$$E_x^T = \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt \tag{2.5}$$

and the average power dissipated by the signal during the interval is

$$P_x^T = \frac{1}{T}E_x^T = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt \qquad \text{potom } E = P.T \tag{2.6}$$

A signal is an energy signal if, and only if, it has nonzero but finite energy for all time:

$$E_x = \lim_{T \to \infty} \int_{T/2}^{T/2} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt \qquad (0 < E_x < \infty)$$

A signal is a power signal if, and only if, it has finite but nonzero power for all time:

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{T/2}^{T/2} |x(t)|^2 dt \qquad (0 < P_x < \infty)$$

General rule:

Periodic and random signals are power signals.

Signals that are both deterministic and non-periodic are energy signals.

$$E = P.T$$

The performance of a communication system depends on the received signal energy; higher energy signals are detected more reliably (with fewer errors) than are lower energy signals - the received energy does the work. On the other hand, power is the rate at which energy is delivered. It is important for different reasons. The power determines the voltages that must be applied to a transmitter and the intensities of the electromagnetic fields that one must contend with in radio systems (i.e., fields in waveguides that connect the transmitter to the antenna, and fields around the radiating elements of the antenna).

In analyzing communication signals, it is often desirable to deal with the waveform energy. We classify x(t) as an energy signal if, and only if, it has nonzero but finite energy ($0 < E_x < \infty$) for all time, where

$$E_x^T = \lim_{T\to\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt$$

$$= \int_{-\infty}^{\infty} x^2(t)dt \qquad (2.7)$$

In the real world, we always transmit signals having finite energy ($0 < E_x < \infty$). However, in order to describe periodic signals, which by definition (Equation 2.1) exist for all time and thus have infinite energy, and in order to deal with random signals that have infinite energy, it is convenient to define a class of signals called power signals. A signal is defined as a power signal if, and only if, it has finite but nonzero power ($0 < P_x < \infty$) for all time, where

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt \qquad (2.8)$$

The energy and power classifications are mutually exclusive. An energy signal has finite energy but *zero average power*, whereas a power signal has finite average power but *infinite energy*. A waveform in a system may be constrained in either its power or energy values. As a general rule, periodic signals and random signals are classified as power signals, while signals that are both deterministic and nonperiodic are classified as energy signals.

Signal energy and power are both important parameters in specifying a communication system. The classification of a signal as either an energy signal or a power signal is a convenient model to facilitate the mathematical treatment of various signals and noise.

Energy and Power Signals

In signal analysis, it is customary to assume a 1Ω resistor, so regardless of whether g(t) represents a voltage across it or a current through it, we may express the instantaneous power p(t) associated with the signal g(t) as $p(t) = |g(t)|^2$.

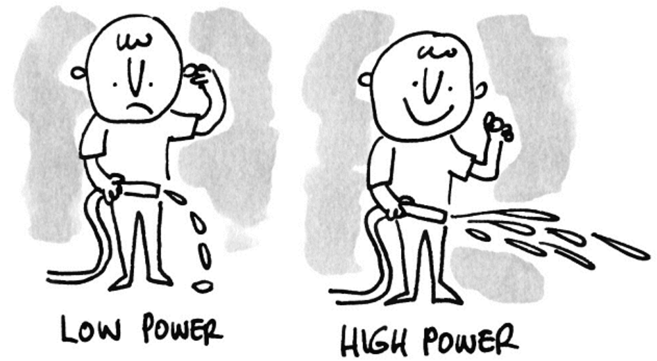
The magnitude squared is used in the instantaneous normalized power to allow the possibility of g(t) being a complex-valued signal. For real signals, we therefore have $p(t) = g^2(t)$.

It is important to highlight that mathematically, power is the derivative of energy with respect to time, and physically, it is the rate at which energy is supplied or consumed.

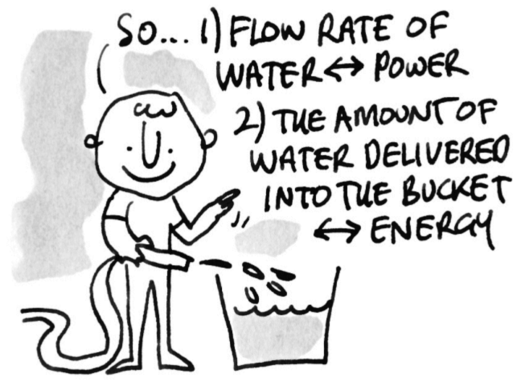
In a digital communication system, power determines the voltage applied to the transmitter, and the system performance directly depends on the received signal energy.

A signal cannot be both an energy signal and a power signal; if it is one, it cannot be the other.

Power is the rate at which energy flows. Electrical power is analogous to the flow rate of water through a hosepipe



Energy is how much electricity has been generated, stored, or consumed over time.



a) Find the average normalized power in the waveform, $x(t) = A\cos 2\pi f_0 t$, using time averaging.

b) Repeat part (a) using the summation of spectral coefficients.

Solution

a) Using Equation (2.13), we have

$$P_x = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} A^2 cos^2 2\pi f_0 t dt$$

$$= \frac{A^2}{2T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} (1 + cos 4\pi f_0 t) dt$$

$$= \frac{A^2}{2T_0}(T_0) = \frac{A^2}{2}$$

Explanation :

$$P = \frac{A^2}{2} \Rightarrow A^2 = 2P = \frac{2E}{T} \quad \text{because } P = \frac{E}{T}$$

and then

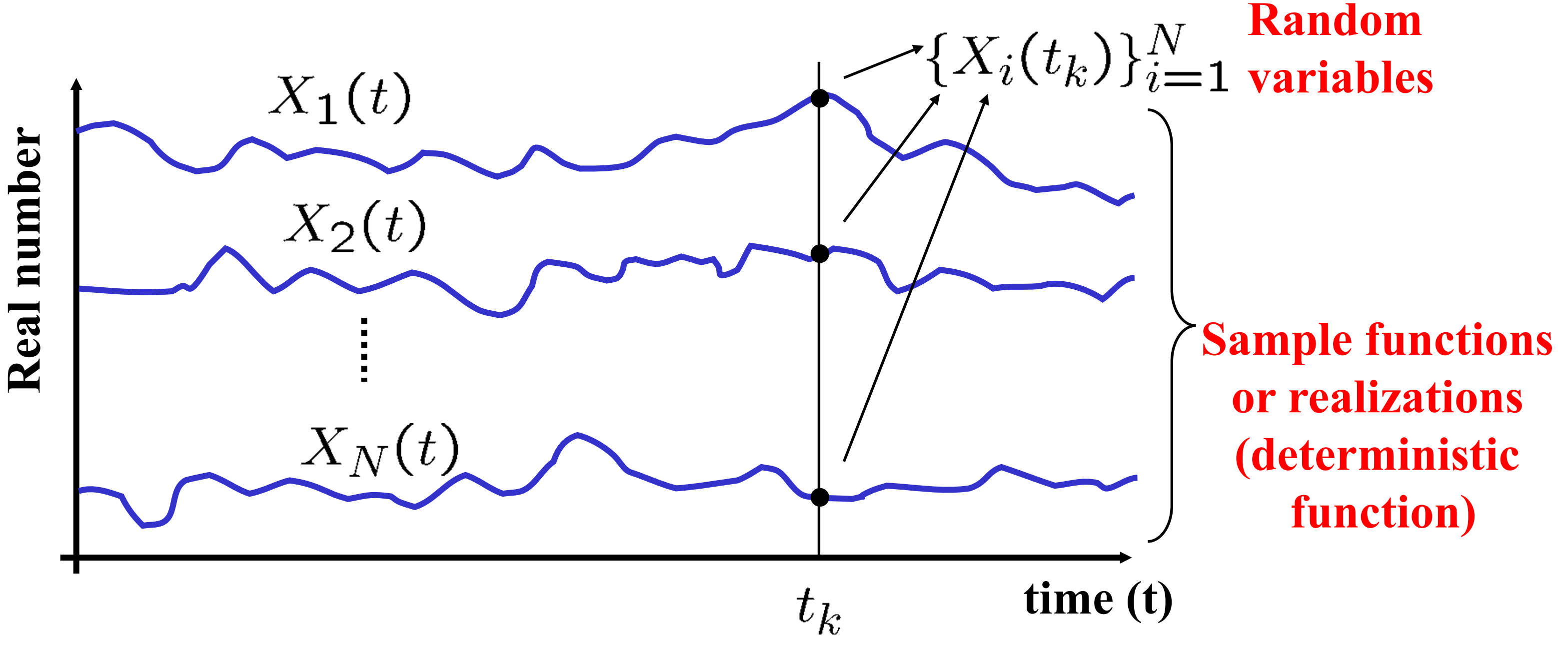
$$A = \sqrt{\frac{2E}{T}}$$

so we can write :

$$x(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_0 t)$$

Random process

- A random process is a collection of time functions, or signals, corresponding to various outcomes of a random experiment. For each outcome, there exists a deterministic function, which is called a sample function or a realization.



Random process …

- Strictly stationary: If none of the statistics of the random process are affected by a shift in the time origin.
- Wide sense stationary (WSS): If the mean and autocorrelation function do not change with a shift in the origin time.
- Cyclostationary: If the mean and autocorrelation function are periodic in time.
- Ergodic process: A random process is ergodic in mean and autocorrelation, if

$$m_X = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} X(t)dt$$

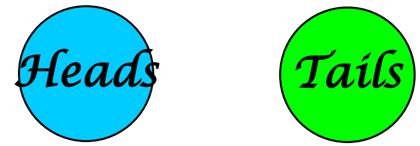
and

$$R_X(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} X(t)X^*(t-\tau)dt$$

, respectively.

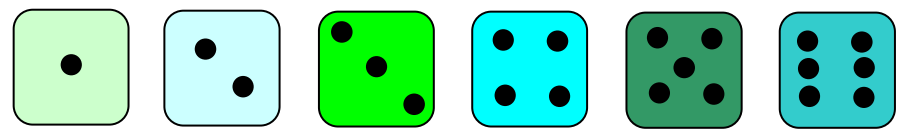
- The set of all possible outcomes
- Tossing a coin

  S = {H,T}



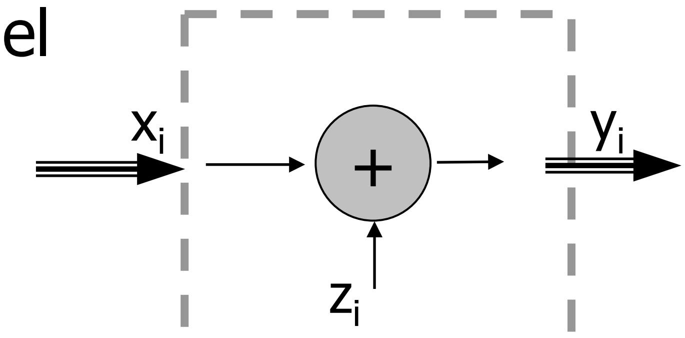
- Rolling a die

  S = {1,2,3,4,5,6}



- The AWGN in a Communication Channel

  $S = [-\infty, \infty]$



Autocorrelation of an energy signal

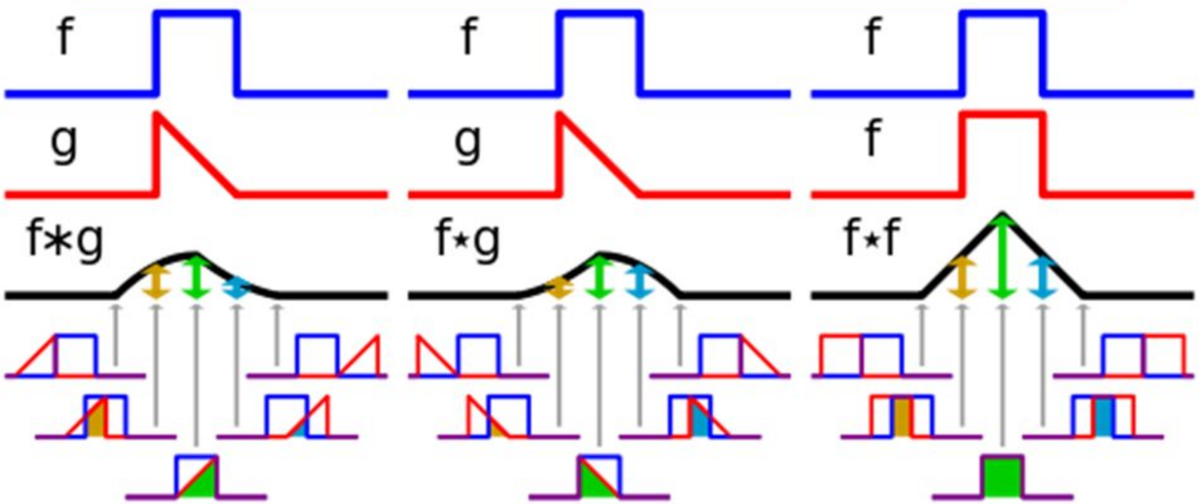
$$R_x(\tau) = x(\tau) \star x^*(-\tau) = \int_{-\infty}^{\infty} x(t)x^*(t-\tau)dt$$

Autocorrelation of a power signal

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x^*(t-\tau)dt$$

- *Autocorrelation and spectral density form a Fourier transform pair.*
- *Autocorrelation is symmetric around zero.*
- *Its maximum value occurs at the origin.*
- *Its value at the origin is equal to the average power or energy.*

Convolution, Cross-correlation, and Autocorrelation



Convolution describes the response of a linear and time-invariant system to an input signal.

The inverse Fourier transform of the pointwise product in frequency space.

Cross-correlation is a measure of similarity of two signals.

It can be used for finding a shift between two signals.

Auto-correlation is the cross-correlation of a signal with itself.

It can be used for finding periodic signals obscured by noise.

http://en.wikipedia.org/wiki/Convolution

Baseband signals

- The simplest signaling scheme is pulse amplitude modulation (PAM)

  With binary PAM a pulse of amplitude A is used to represent a "1" and a pulse with amplitude -A to represent a "0"

- The simplest pulse is a rectangular pulse, but in practice other type of pulses are used

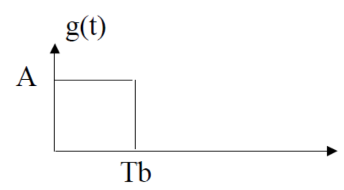
  For our discussion we will usually assume a rectangular pulse

- If we let g(t) be the basic pulse shape, than with PAM

  we transmit g(t) to represent a "1" and -g(t) to represent a "0"

$1 \Rightarrow S(t) = g(t)$

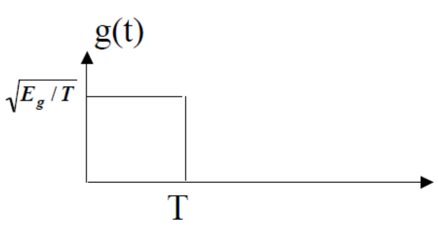
$0 \Rightarrow S(t) = -g(t)$



The signal energy depends on the amplitude
Eg is the energy of the signal pulse g(t)
For rectangular pulse with energy Eg =>

$$E_g = \int_0^T A^2 dt = TA^2 \Rightarrow A = \sqrt{E_g / T}$$

$$g(t) = \begin{cases} \sqrt{E_g / T} & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

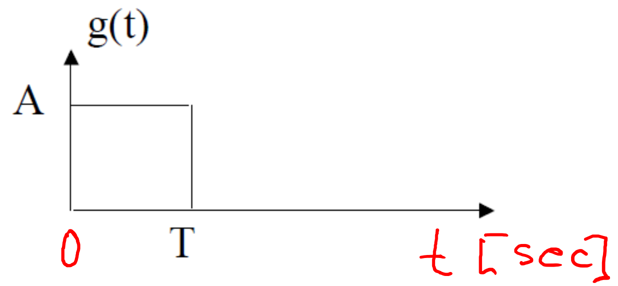


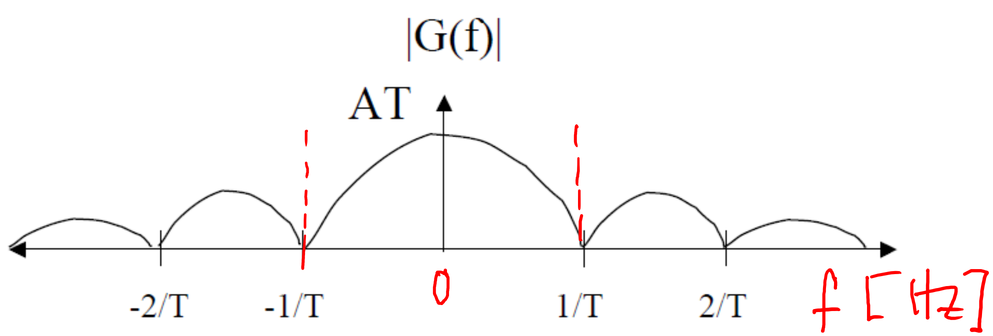
Bandwidth occupancy (ideal rectangular pulse)

G(f) = F[g(t)]

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt = \int_{0}^{T} A e^{-j2\pi ft} dt$$

$$G(f) = (AT) Sinc(\pi f T) e^{-j\pi f T}$$





- **Ideal rectangular pulse has unlimited bandwidth**
  - **First "null" bandwidth = 2(1/T) = 2/T**
- **In practice, we "shape" the pulse so that most of its energy is contained within a small bandwidth**

(1) Figure 1 is the illustration of the Fourier Transform pair in regard to a Rectangular pulse in the time domain and a Sinc function in the frequency domain. We want to observe the physical symptoms of the impact of time duration of rectangular pulse on the effective bandwidth and peak point.

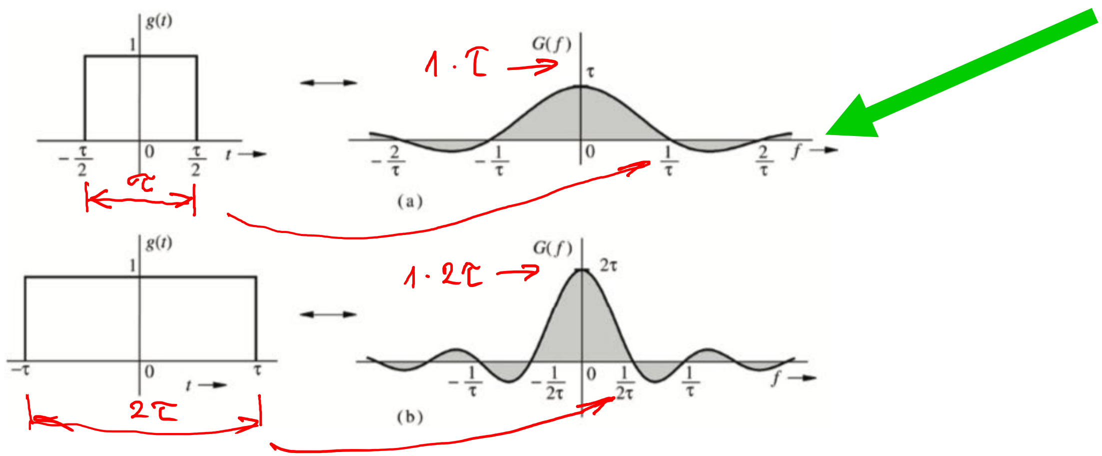
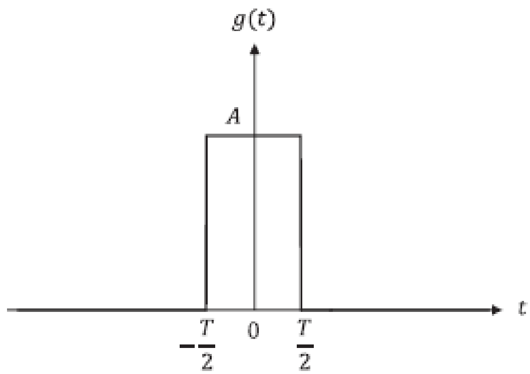


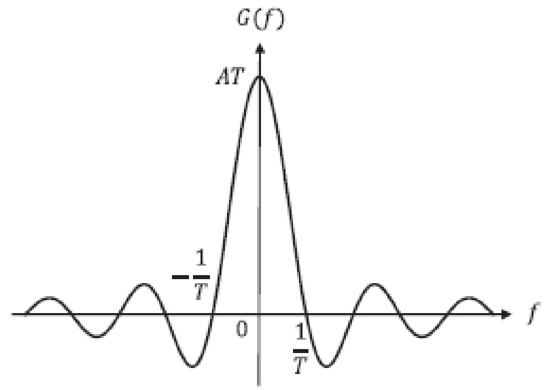
Fig. 1: Fourier Transform pair: Rectangular pulse and Sinc function

(a) Mathematically derive the fourier transform of rectangular pulse described in Fig. 1-(a). It is in general notated as $g(t) = \text{rect}(t/\tau) = \sqcap(t/\tau)$.

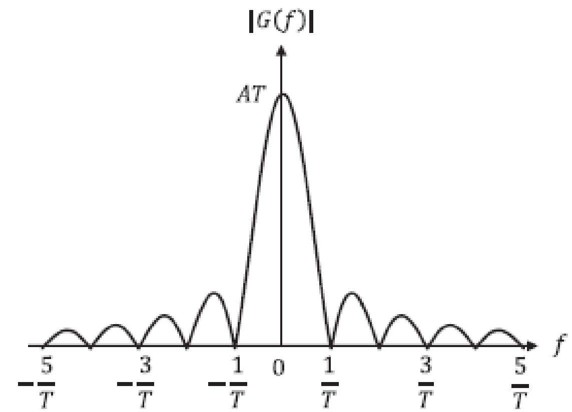
(b) Draw magnitude plot and phase plot of the spectra (Fourier Transform) for $\tau = 10$, $20, 40$ [seconds]. In other words, overlap three magnitude curves for $\tau = 10, 20, 40$ in one plot; and overlap three phase curves for $\tau = 10, 20, 40$ in the other plot.



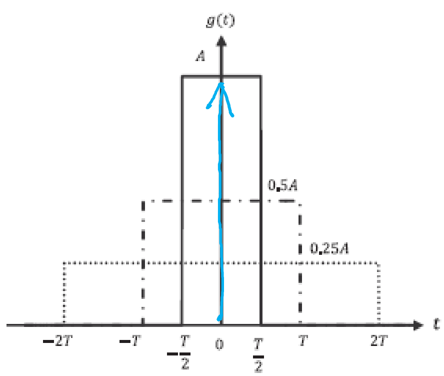
g(t)
A
t
−T/2
0
T/2



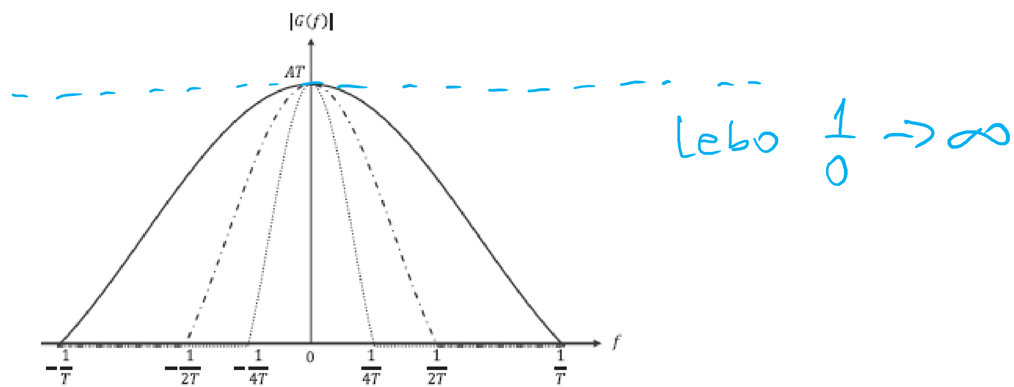
G(f)
AT
−1/T
0
1/T
f



|G(f)|
AT
f
−5/T
−3/T
−1/T
0
1/T
3/T
5/T

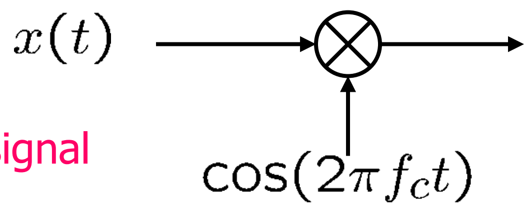


g(t)
A
0.5A
0.25A
t
−2T
−T
−T/2
0
T/2
T
2T



|G(f)|
AT
f
−1/T
−1/2T
−1/4T
0
1/4T
1/2T
1/T
lebo 1/0 → ∞

Baseband versus bandpass signals/spectra:

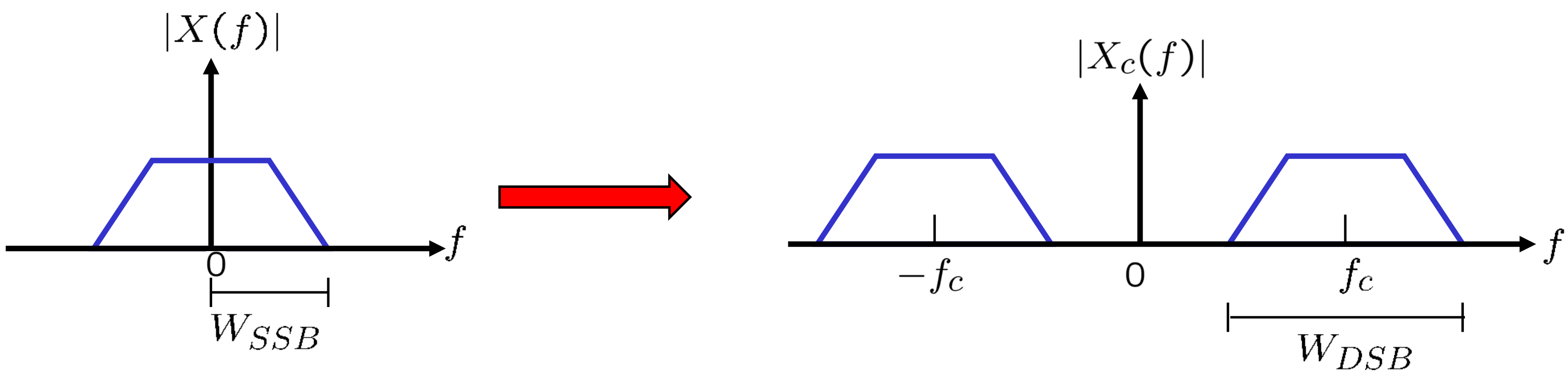


$$x_c(t) = x(t)\cos(2\pi f_c t)$$

Baseband signal

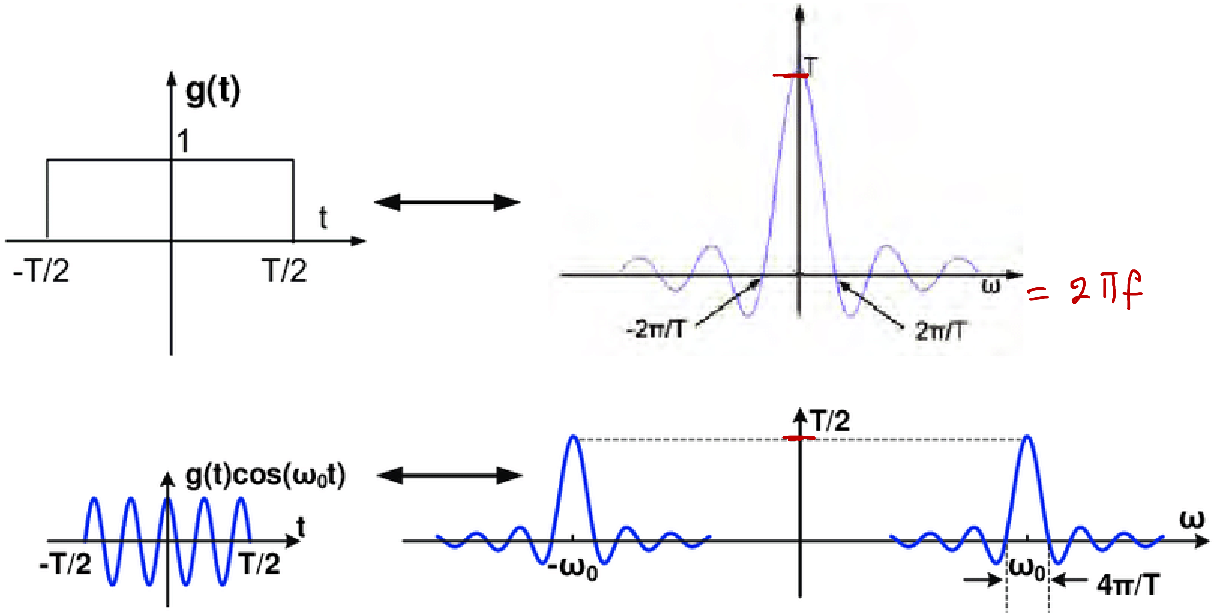
Local oscillator

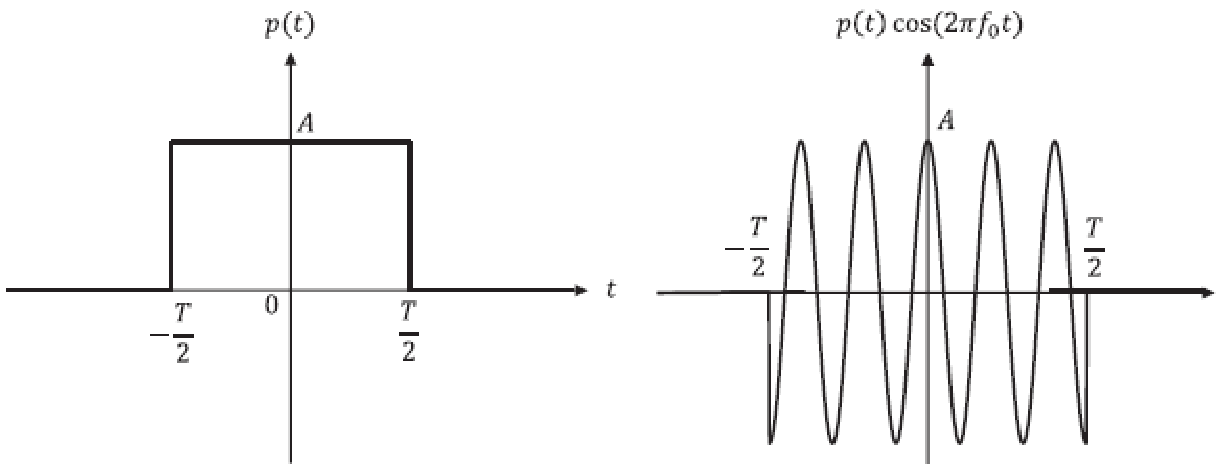
Bandpass signal



Bandwidth dilemma:

Bandlimited signals are not realizable!
Realizable signals have infinite bandwidth!

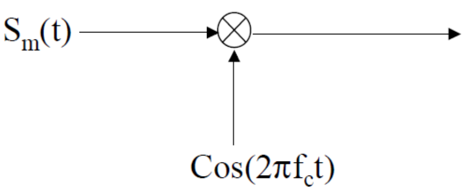

g(t)
1
t
-T/2
T/2
T
ω
= 2πf
-2π/T
2π/T
g(t)cos(ω0t)
t
-T/2
T/2
T/2
ω
-ω0
ω0
4π/T


p(t)
A
0
−T/2
T/2
t
p(t) cos(2πf0t)
A
−T/2
T/2
t

(a) (b)

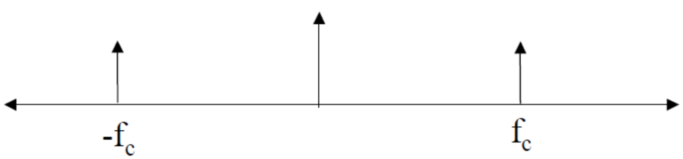
Bandpass signals

To transmit a baseband signal S(t) through a pass-band channel at some center frequency f_c, we multiply S(t) by a sinusoid with that frequency

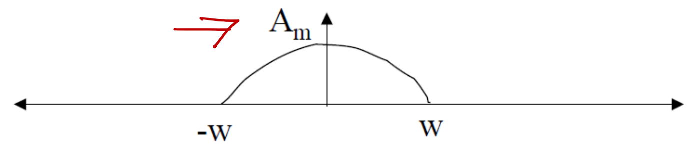


$$U_m(t) = S_m(t) Cos(2\pi f_c t) = A_m g(t)\ Cos(2\pi f_c t)$$

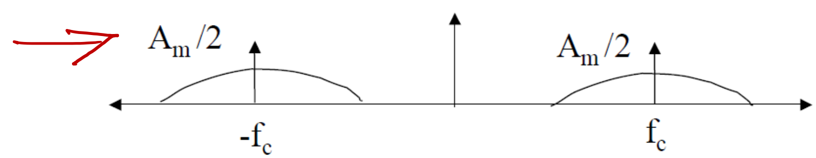
$$F[Cos(2\pi f_c t)] = (\delta(f-f_c)+\delta(f+f_c))/2$$



$F[A_m g(t)]$ = depends on g()



$F[A_m g(t)\ Cos(2\pi f_c t)]$



Recall: Multiplication in time = convolution in frequency

Energy content of modulated signals

$$E_m = \int_{-\infty}^{\infty} U_m^2(t)dt = \int_{-\infty}^{\infty} A_m^2 g^2(t) Cos^2(2\pi f_c t)dt$$

$$Cos^2(\alpha) = \frac{1+\cos(2\alpha)}{2}$$

$$E_m = \frac{A_m^2}{2}\int_{-\infty}^{\infty} g^2(t) + \underbrace{\frac{A_m^2}{2}\int_{-\infty}^{\infty} g^2(t) Cos(4\pi f_c t)dt}_{\approx 0}$$

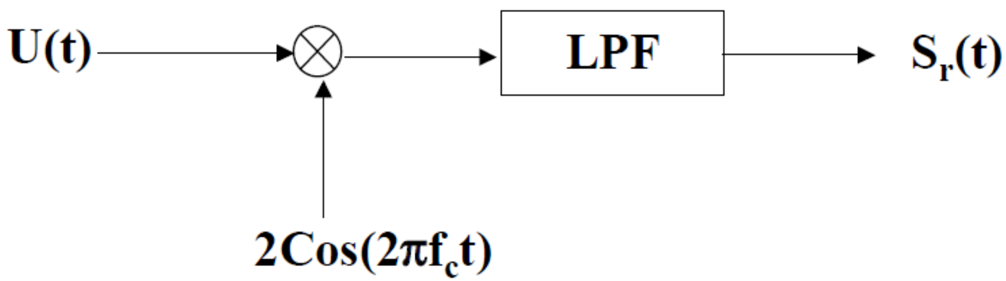
$$E_m = \frac{A_m^2}{2} E_g$$

- **The cosine part is fast varying and integrates to 0**
- **Modulated signal has 1/2 the energy as the baseband signal**

Demodulation

- To recover the original signal, multiply the received signal ($U_m(t)$) by a cosine at the same frequency

$$U_m(t) = S_m(t)Cos(2\pi f_c t) = A_m g(t)\ Cos(2\pi f_c t)$$

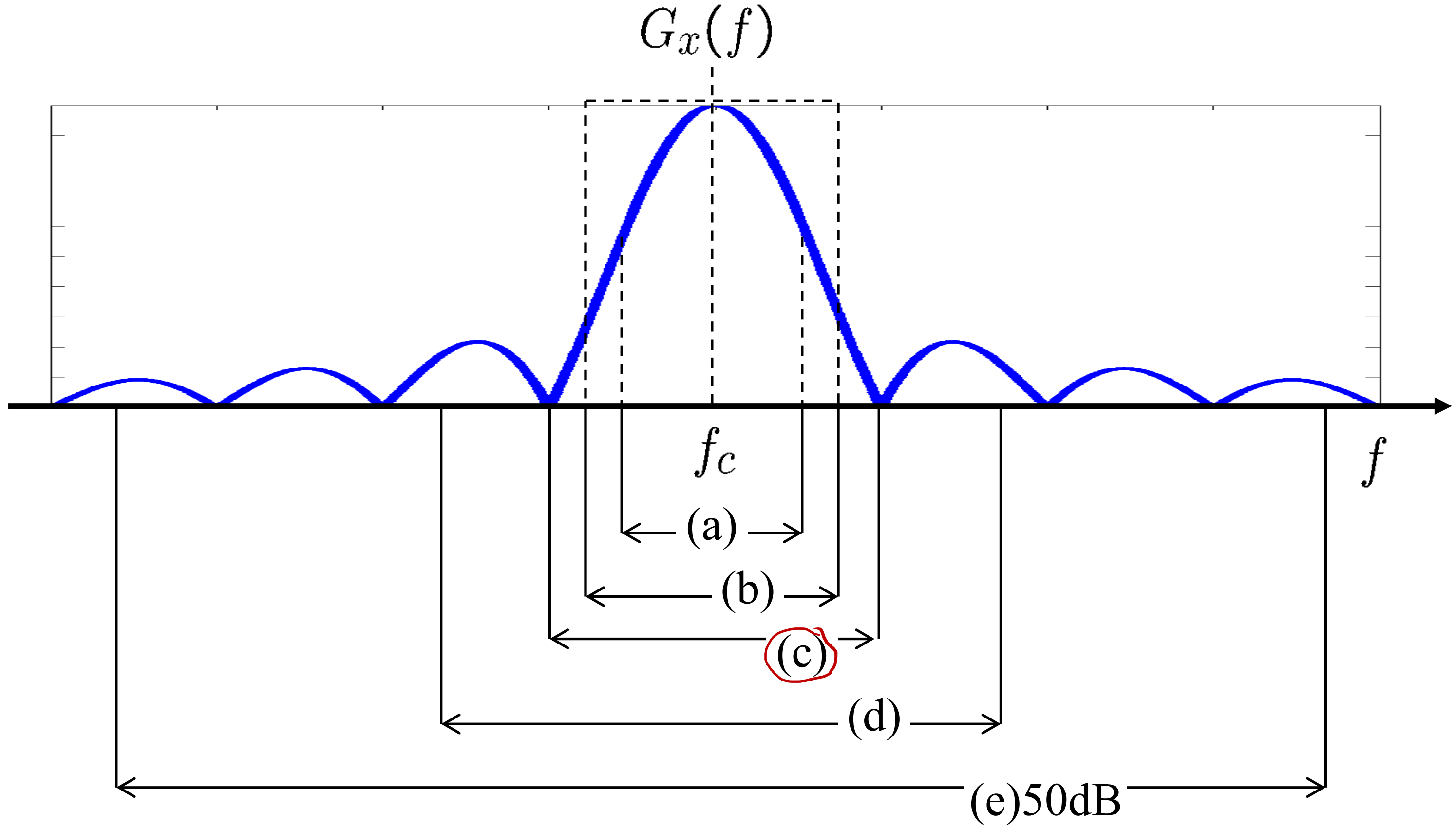


$$U(t)2Cos(2\pi f_c t) = 2S(t)Cos^2(2\pi f_c t) = S(t) + S(t)Cos(4\pi f_c t)$$

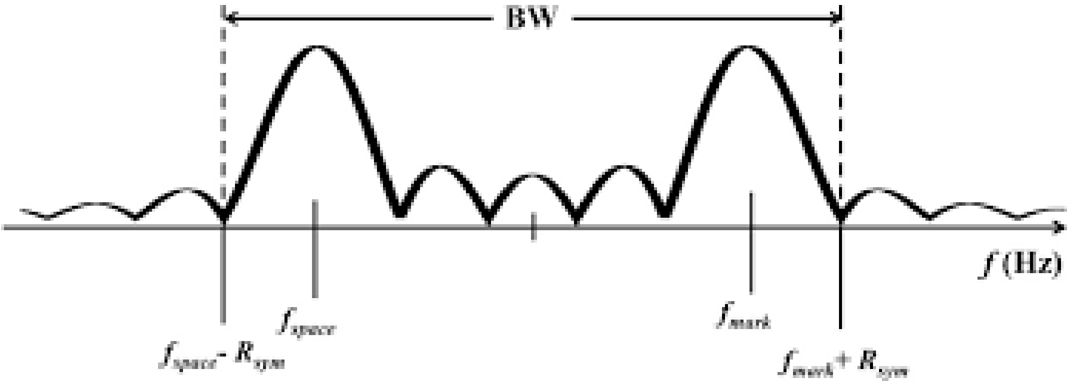
- The high frequency component is rejected by the LPF and we are left with S(t)

a) Half-power bandwidth
b) Noise equivalent bandwidth
c) Null-to-null bandwidth
d) Fractional power containment bandwidth
e) Bounded power spectral density
f) Absolute bandwidth

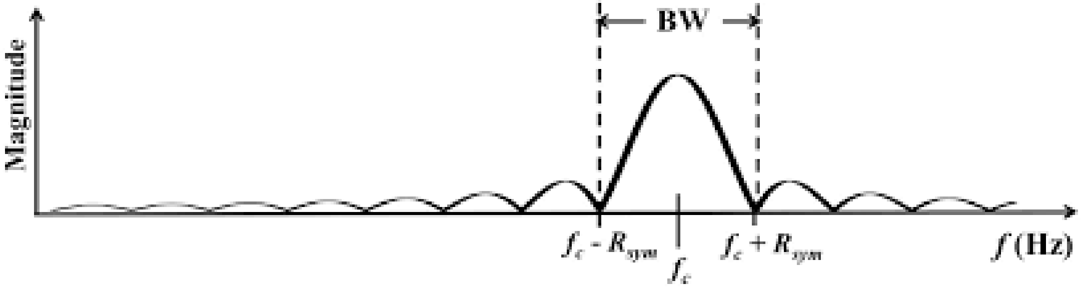
(a) Half-power bandwidth. This is the interval between frequencies at which Gx(f) has dropped to half-power, or 3 dB below the peak value.
(c) Null-to-null bandwidth. The most popular measure of bandwidth for digital communications is the width of the main spectral lobe, where most of the signal power is contained.



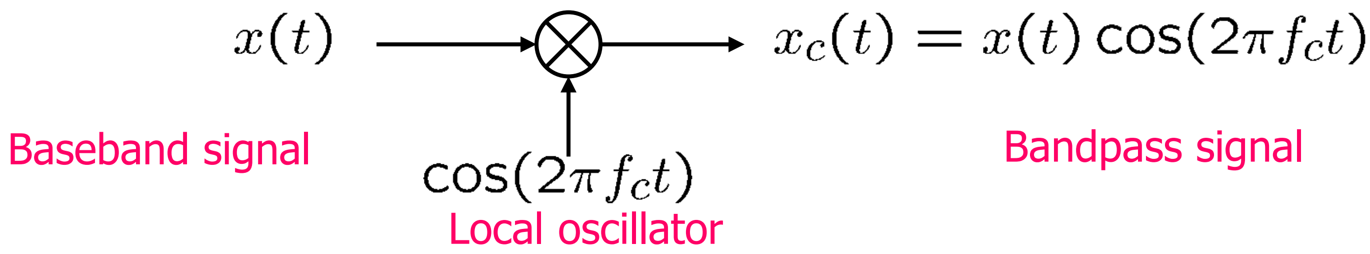
FSK spectrum

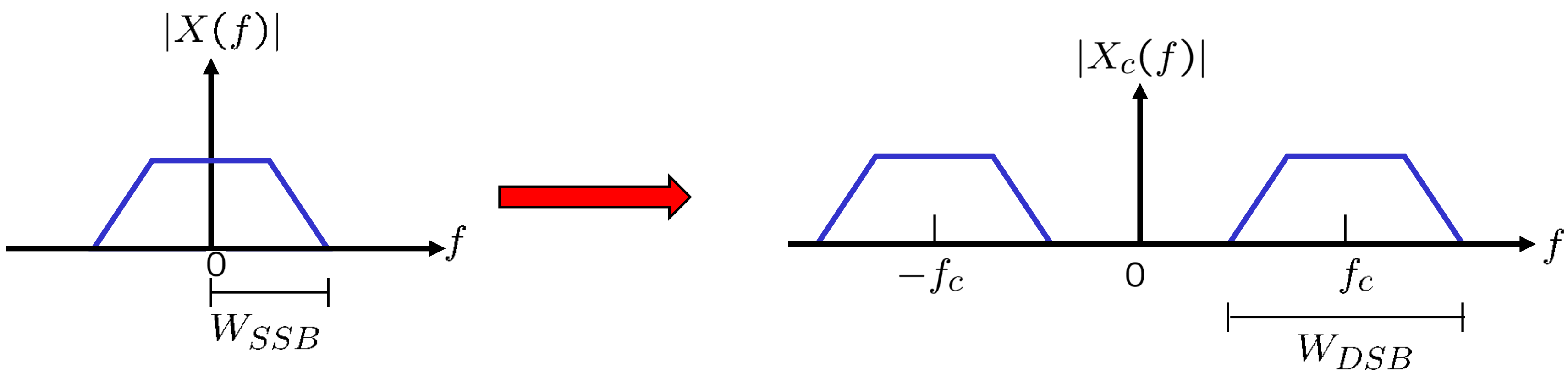


ASK, PSK spectrum



Baseband versus bandpass signals/spectra:





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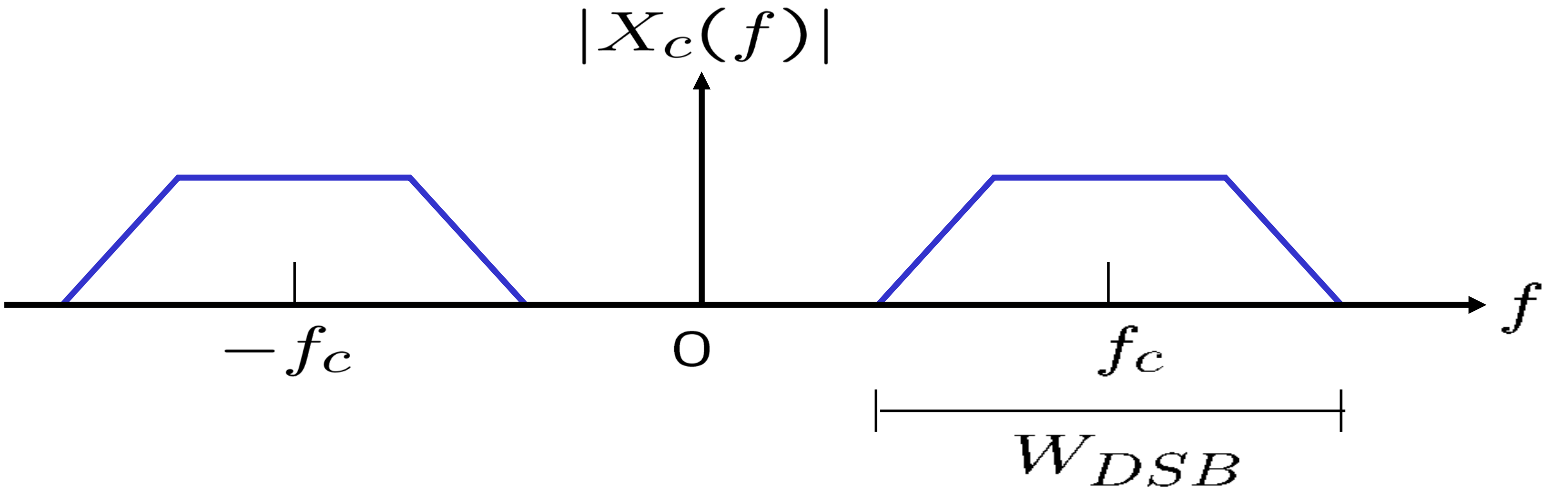
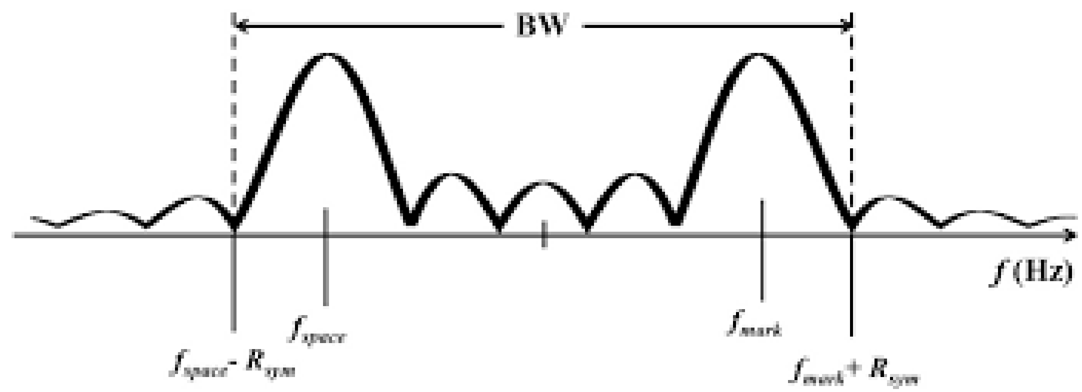


FSK spectrum



ASK, PSK spectrum