3.1 PROBABILITY

The sample space – S (a die) of the experiment consists of the set of all possible outcomes. In the case of the die

$$S = \{1,2,3,4,5,6\} \quad (3.1)$$

where the integers 1, …, 6 represent the number of dots on the six faces of the die. These six possible outcomes are the sample points of the experiment. An event is a subset of S and may consist of any number of sample points. For example, the event A defined as

$$A = \{2,4\} \quad (3.2)$$

consists of the outcomes 2 and 4. The complement of the event A, denoted by $\bar{A}$, consists of all the sample points in S that are not in A and, hence

$$\bar{A} = \{1,3,5,6\} \quad (3.3)$$

Two events are said to be *mutually exclusive* if they have no sample points in common – that is, if the occurrence of one event excludes the occurrence of the other. For example, if A is defined as in Equation 3.2 and the event B is defined as

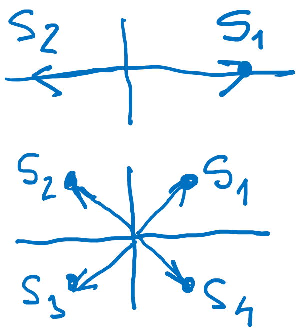
$$B = \{1,3,6\} \quad (3.4)$$

then A and B are mutually exclusive events. In this case we can write

$$P(A) + P(B) = \frac{2}{6} + \frac{3}{6} = \frac{5}{6} \quad (3.5)$$

$M=2; \quad S=\{s_1, s_2\}$

$M=4; \quad S=\{s_1, s_2, s_3, s_4\}$



pre rovnakú pravdepod. výskytu symbolov $s_1, \dots s_4$

a) $P(s_1) = P(s_2) = 1/2$

b) $P(s_1) = P(s_2) = P(s_3) = P(s_4) = 1/4$

3.1.1 Joint Events and Joint Probabilities

Instead of dealing with a single experiment, let us perform two experiments and consider their outcomes. For example, the two experiments may be two separate tosses of a single die or a single toss of two dice. In either case, the sample space S consists of the 36 two - tuples (i,j) where $i, j = 1,2, \dots, 6$. If the dice are fair, each point in the sample space is assigned the probability $1/36$.

In general, if one experiment has the possible outcomes $A_i, i = 1,2, \dots, n$, and the second experiment has the possible outcomes $B_j, j = 1,2, \dots, m$, then the combined experiment has the possible outcomes $(A_i, B_j), i = 1,2, \dots, n, j = 1,2, \dots, m$. Associated with each joint outcome (A_i, B_j) is the ***joint probability*** $P(A_i, B_j)$which satisfies the condition

$$0 \leq P(A_i, B_j) \leq 1 \quad (3.6)$$

Assuming that the outcomes B_j are mutually exclusive, it follows that

$$\sum_{j=1}^{m} P(A_i, B_j) = P(A_i) \quad (3.7)$$

Similarly, if outcomes A_i, are mutually exclusive then

$$\sum_{i=1}^{n} P(A_i, B_j) = P(B_j) \quad (3.8)$$

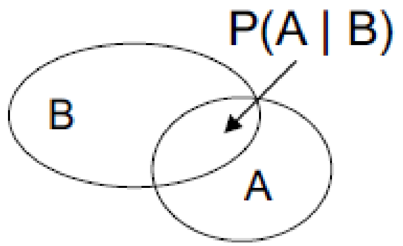
Furthermore, if all the outcomes of the two experiments are mutually exclusive, then

$$\sum_{i=1}^{n} \sum_{j=1}^{m} P(A_i, B_j) = 1 \quad (3.9)$$

3.1.2 *Conditional Probabilities*

Consider a combined experiment in which a joint event occurs with probability $P(A,B)$. Suppose that the event B has occurred and we wish to determine the probability of occurrence of the event A. The conditional probability of the event A given the occurrence of the event B is defined as

$$P(A|B) = \frac{P(A,B)}{P(B)} \tag{3.10}$$



In a similar manner

$$P(B|A) = \frac{P(A,B)}{P(A)} \tag{3.11}$$

The relations in Equation 3.10 and 3.11 may also be expressed as

$$P(A,B) = P(A|B)P(B) = P(B|A)P(A) \tag{3.12}$$

An extremely useful relationship for conditional probabilities is ***Bayes' theorem***.

$$P(A_i|\mathrm{B}) = \frac{P(A_i,B)}{P(B)}$$

$$= \frac{P(B|A_i)P(A_i)}{\sum_{j=1}^{n} P(B|A_j)P(A_j)} \quad (3.13)$$

We use this formula to derive the structure of the optimum receiver for a digital communication system in which:

- the events A_i, i=1,2…,n, represent the possible transmitted messages in a given time interval;
- $P(A_i)$ represent their a priori probabilities;
- B represents the received signal, which consists of the transmitted message (one of the A_i) corrupted by noise;
- and $P(A_i|B)$ is the a posteriori probability of A_i conditioned on having observed the received signal B.

3.1.3 Statistical Independence

The statistical independence of two or more events is another concept in probability theory. It usually arises when we consider two or more experiments or repeated trials of a single experiment. To explain this concept, we consider two events A and B and their conditional probability $P(A|B)$, which is the probability of occurrence of A given that B has occurred. Suppose that the occurrence of A does not depend on the occurrence of B. That is

$$P(A|\mathrm{B}) = P(A) \quad (3.14)$$

After substitution

$$P(A,\mathrm{B}) = P(A)P(B) \quad (3.15)$$

When the events A and B satisfy the relation in Equation 3.15, they are said to be *statistically independent*.

An extremely useful relationship for conditional probabilities is ***Bayes' theorem***.

ak n=2 potom

$$P(A_1|B) = \frac{P(B|A_1)P(A_1)}{P(B|A_1)P(A_1) + P(B|A_2)P(A_2)}$$

$$P(A_2|B) = \frac{P(B|A_2)P(A_2)}{P(B|A_1)P(A_1) + P(B|A_2)P(A_2)}$$

$$P(A_i|\mathrm{B}) = \frac{P(A_i,B)}{P(B)}$$

$$= \frac{P(B|A_i)P(A_i)}{\sum_{j=1}^{n} P(B|A_j)P(A_j)} \tag{3.13}$$

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In a binary communication system, the symbols 0 and 1 are sent through a channel, but noise can switch 1 to 0 or vice versa with certain probabilities. The error can be estimated by Bayes' rule

Aká je pravdepodobnosť, že bol vyslaný symbol A_i , ak sme prijali symbol B

Kanálová pravdepodobnosť vstup/výstup

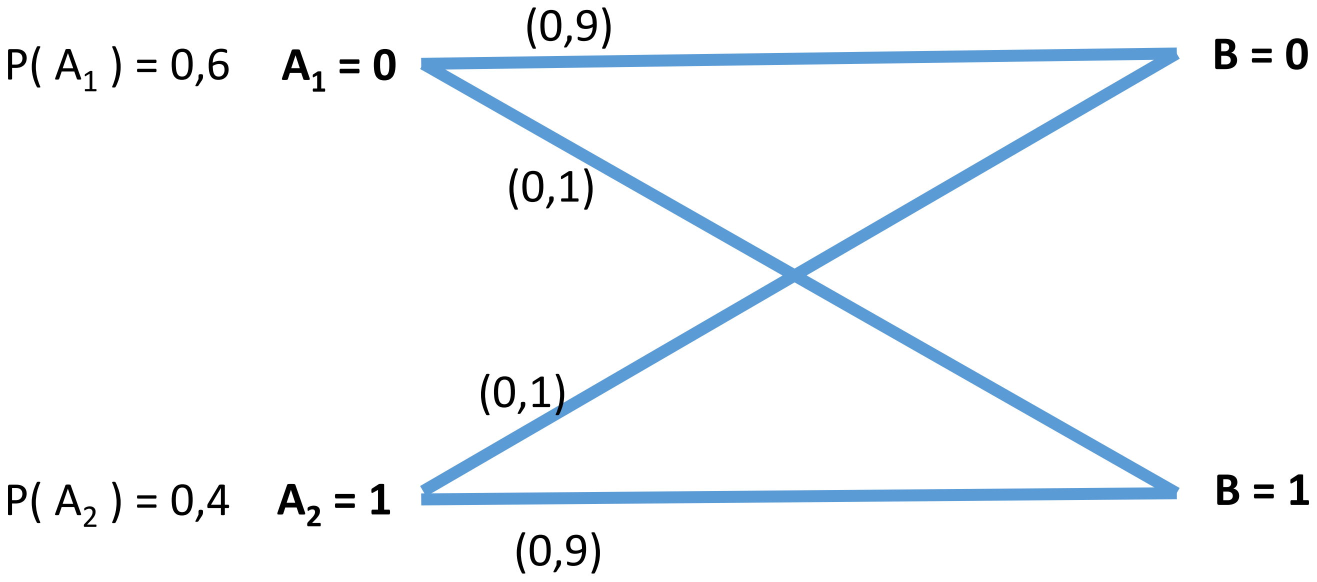
Apriórna pravdepodobnosť symbolu na vstupe

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{j=1}^{n} P(B|A_j)P(A_j)}$$

ak: n = 2

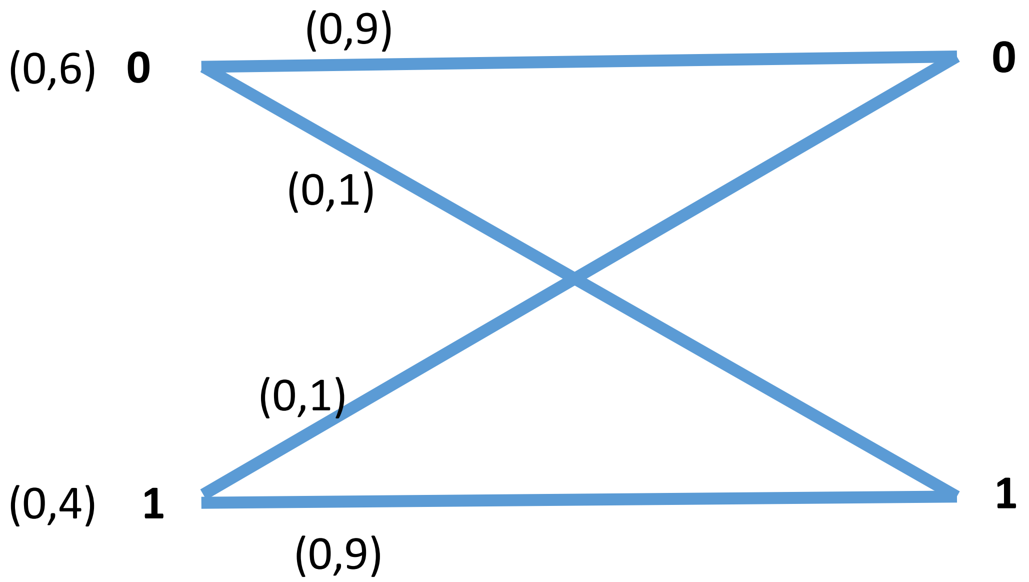
$$P(A_1|B) = \frac{P(B|A_1)P(A_1)}{P(B|A_1)P(A_1) + P(B|A_2)P(A_2)}$$

$$P(A_2|B) = \frac{P(B|A_2)P(A_2)}{P(B|A_1)P(A_1) + P(B|A_2)P(A_2)}$$



$$\mathrm{P}(0|0) = \frac{0{,}9\ x\ 0{,}6}{0{,}9x0{,}6+0{,}1x0{,}4} = \frac{0{,}54}{0{,}58} = 0{,}931$$

A digital communication system sends a sequence of 0 and 1, each of which are received at the other end of a link. Assume that the probability that 0 is received correctly is 0.90 and that a 1 is received correctly is 0.90. Alternately, the probability that a 0 or 1 is not received correctly is 0.10. The probability that a 0 is sent is 60% and that a one is sent is 40%.



a) What is the probability that a received 0 was transmitted as a 0?

$$P(0|0) = \frac{0{,}9 \; x \; 0{,}6}{0{,}9x0{,}6+0{,}1x0{,}4} = \frac{0{,}54}{0{,}58} = 0{,}931$$

b) What is the probability that a received 1 was transmitted as a 1?

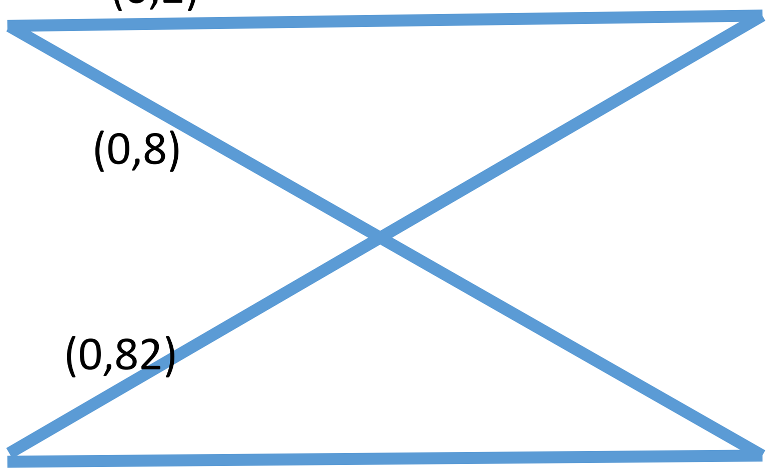
$$P(1|1) = \frac{0{,}9 \; x \; 0{,}4}{0{,}9x0{,}4+0{,}1x0{,}6} = \frac{0{,}36}{0{,}42} = 0{,}857$$

c) What is the probability that a received 0 was transmitted as a 1?

$$P(1|0) = \frac{0{,}1 \; x \; 0{,}4}{0{,}1x0{,}4+0{,}9x0{,}6} = \frac{0{,}04}{0{,}58} = 0{,}069$$

d) What is the probability that a received 1 was transmitted as a 0?

$$P(0|1) = \frac{0{,}6 \; x \; 0{,}1}{0{,}6x0{,}1+0{,}4x0{,}9} = \frac{0{,}06}{0{,}42} = 0{,}143$$



$$P(0|0) = \frac{0,2\ x\ 0,15}{0,2x0,15+0,85x0,82} = 0,041 = 4,1\ \%$$

$$P(1|0) = \frac{0,82\ x\ 0,85}{0,82x0,85+0,2x0,15} = 0,958$$

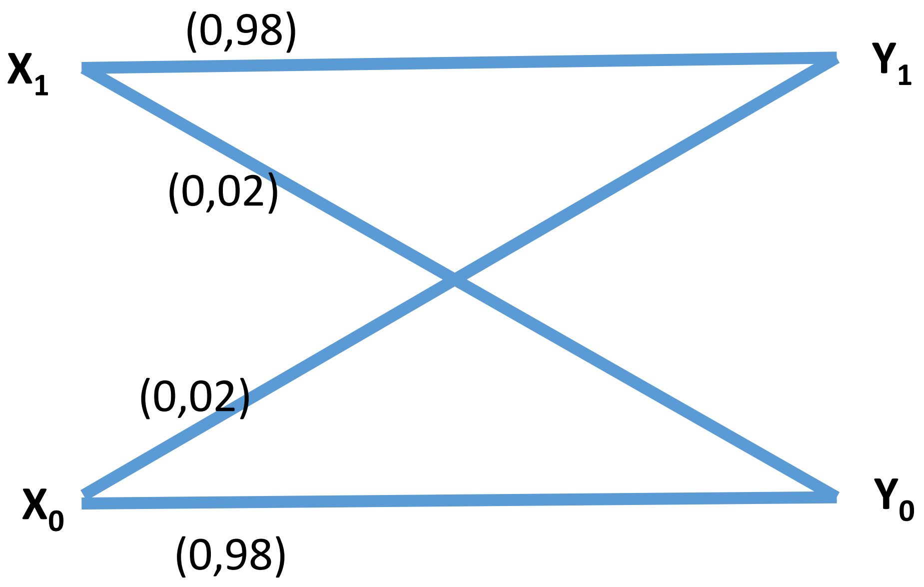
$$P(0|1) = \frac{0,8\ x\ 0,15}{0,8x0,15+0,18x0,85} = 0,439$$

$$P(1|1) = \frac{0,18\ x\ 0,85}{0,18x0,85+0,15x0,8} = 0,56$$

BSC is the simplest model for information transmission via a discrete channel (channel is ideal, no amplitude and phase distortion, only distortion is due to AWGN):

$P(X_1) = 0{,}7$

$P(X_0) = 0{,}3$



$P(X_1 | Y_1) = 0{,}9913$

$P(X_0 | Y_1) = 0{,}0087$

$P(X_1 | Y_0) = 0{,}0455$

$P(X_0 | Y_0) = 0{,}9545$

Channel's error probability $p_e = 0.02$: on average, bit error rate is 2%

Probability density function – PDF of the random variable X

$$p(x) = \frac{dF(x)}{d(x)} \quad -\infty \leq x \leq \infty \tag{3.18}$$

or, equivalently

$$F(x) = \int_{-\infty}^{x} p(u)du \quad -\infty \leq x \leq \infty \tag{3.19}$$

The discrete part of $p(x)$ may be expressed as

$$p(x) = \sum_{i=1}^{n} P(X = x_i)\delta(x - x_i) \quad -\infty \leq x \leq \infty \tag{3.20}$$

Often we face with the problem of determining the probability that a random variable X falls in an interval (x_1, x_2), where $x_2 > x_1$.

$$P(x_1 < X \leq x_2) = F(x_2) - F(x_1) = \int_{x_1}^{x_2} p(x)dx \tag{3.21}$$

In other words, the probability of the event $\{x_1 < X < x_2\}$ is simply the area under the PDF in the range $x_1 < X \leq x_2$.

3.2.2 Statistical Averages of Random Variables

Averages play an important role in the characterization of the outcomes of experiments and the random variables defined on the sample space of the experiments. Of particular interest are:

- the first and second moments of a single random variable,
- the joint moments, such as the correlation and covariance, between any pair of random variables in a multidimensional set of random variables.

The mean or expected value of X (single random variable) – ***the first moment*** – is

$$E(X) \equiv m_x = \int_{-\infty}^{\infty} x p(x) dx \tag{3.23}$$

where E(X) denotes expectation (statistical averaging).

In general, the ***n^{th} moment*** is defined as

$$E(X^n) = \int_{-\infty}^{\infty} x^n p(x) dx \tag{3.24}$$

Now, suppose that we define a random variable $Y = g(X)$, where $g(X)$ is some arbitrary function of the random variable X. The expected value of Y is

$$E(Y) = E[g(X)] = \int_{-\infty}^{\infty} g(x)p(x)dx \tag{3.25}$$

In particular, if $Y = (X - m_x)^n$, then

$$E(Y) = E[(X - m_x)^n] = \int_{-\infty}^{\infty} (x - m_x)^n p(x)dx \tag{3.26}$$

This expected value is called ***n**^th^ **central moment*** of the random variable X.

When n=2, the central moment is called – ***variance***

$$\sigma_x^2 = \int_{-\infty}^{\infty} (x - m_x)^2 p(x)dx \tag{3.27}$$

This parameter provides a measure of the dispersion of the random variable X.

$$\sigma_x^2 = E[X^2] - E[X]^2 = E(X^2) - m_x^2 \tag{3.28}$$

The ***joint moment*** of X_1 and X_2 is

$$E\left[X_1^k X_2^n\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1^k\, x_2^n p(x_1\,, x_2)dx_1 dx_2 \tag{3.29}$$

The ***joint central moment*** is

$$E[(X_1 - m_1)^k (X_2 - m_2)^n] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x_1 - m_1)^k\,(x_2 - m_2)^n p(x_1\,, x_2)dx_1 dx_2 \tag{3.30}$$

Of particular importance to us are the joint moment and joint central moment corresponding to $k = n = 1$.

These joint moments are called the ***correlation*** and the ***covariance*** of the random variables X_1 and X_2, respectively.

Gaussian (normal) distribution

The PDF is

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp[-\frac{1}{2}(\frac{x-m_x}{\sigma})^2] \quad (3.40)$$

Where m_x is the mean and σ^2 is the variance of the random variable.

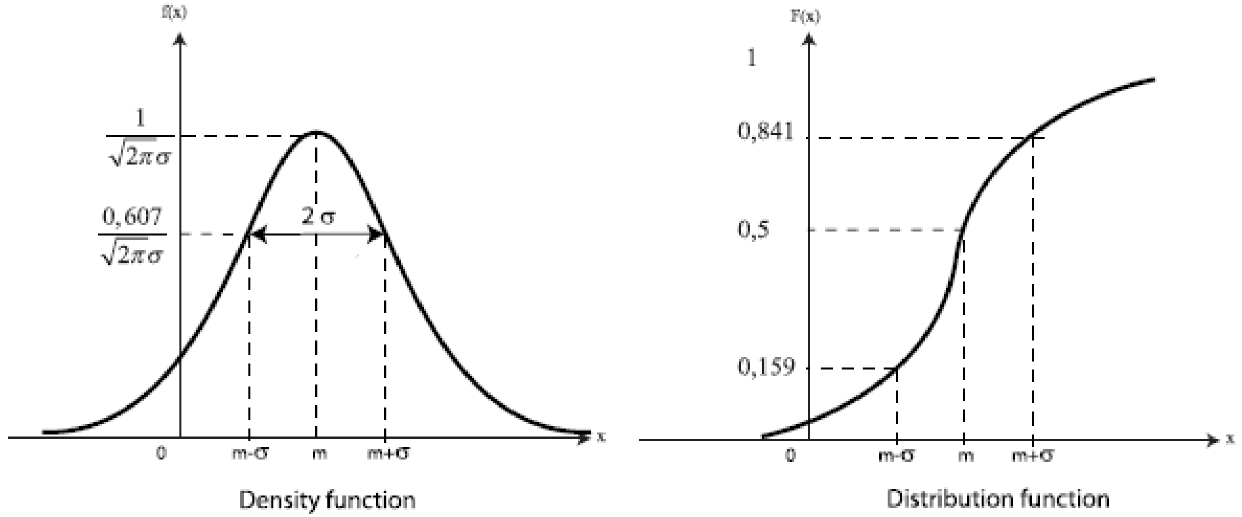


Figure 3.5 The PDF and CDF of a Gaussian random variable

The CDF is

$$F(x) = \int_{-\infty}^{x} p(u)du = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-(u-m_x)^2}{2\sigma^2}} du$$

$$= \frac{1}{2}\frac{2}{\sqrt{\pi}} \int_{-\infty}^{\frac{x-m_x}{\sqrt{2}\sigma}} e^{-t^2} dt = \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{x-m_x}{\sqrt{2}\sigma}\right) \quad (3.41)$$

where erf(x) denotes the *error function*, defined as

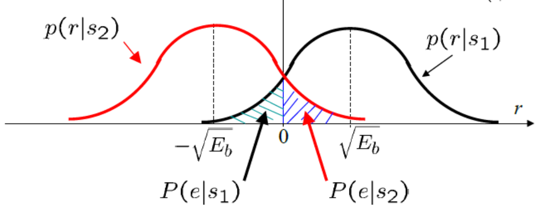
$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} dt \quad (3.42)$$

The function that is frequently used for the area under the tail of the Gaussian PDF is denoted by *Q(x)* and defined as

$$\mathrm{Q}(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{\frac{-t^2}{2}} dt \quad (3.43)$$

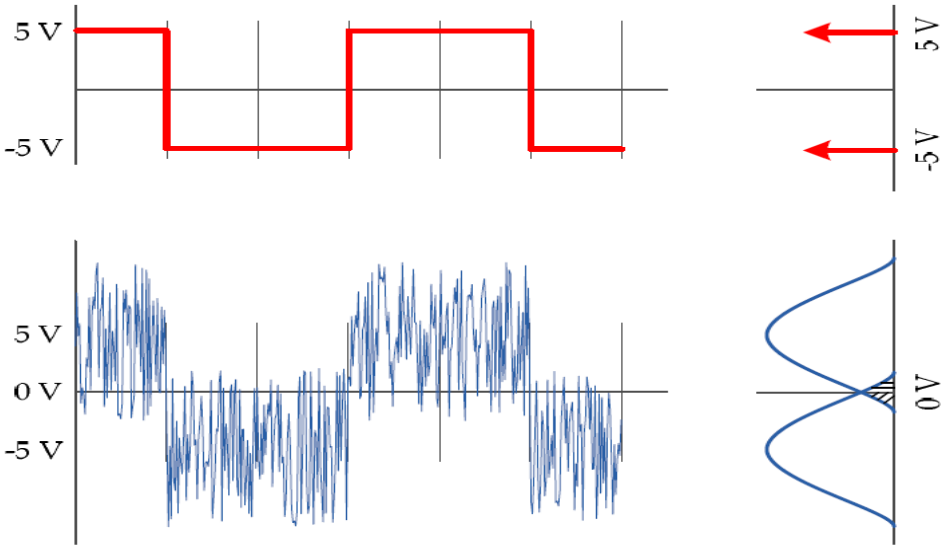
$$\mathrm{Q}(x) = \frac{1}{2}\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right) \quad (3.44)$$

Due to symmetry $P(e|s_2) = P(r > 0|s_2) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

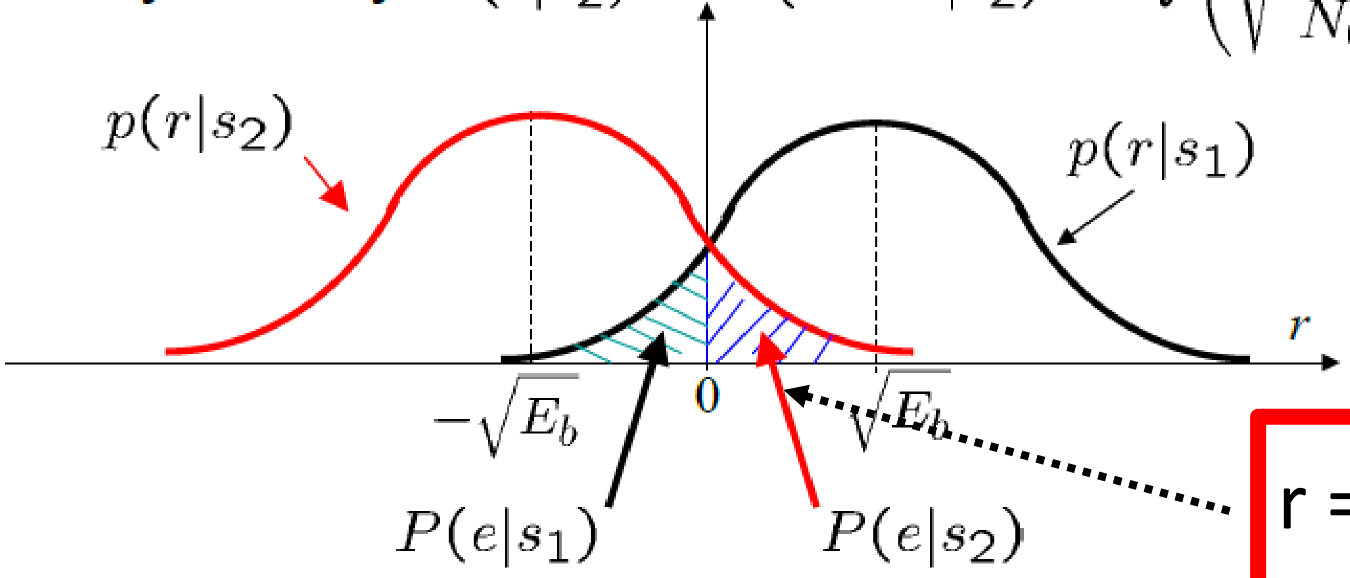


$$P_{\mathrm{BER-BPSK(coherent)}} = \mathrm{Q}\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$P_{\mathrm{BER-BASK(coherent)}} = \mathrm{Q}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

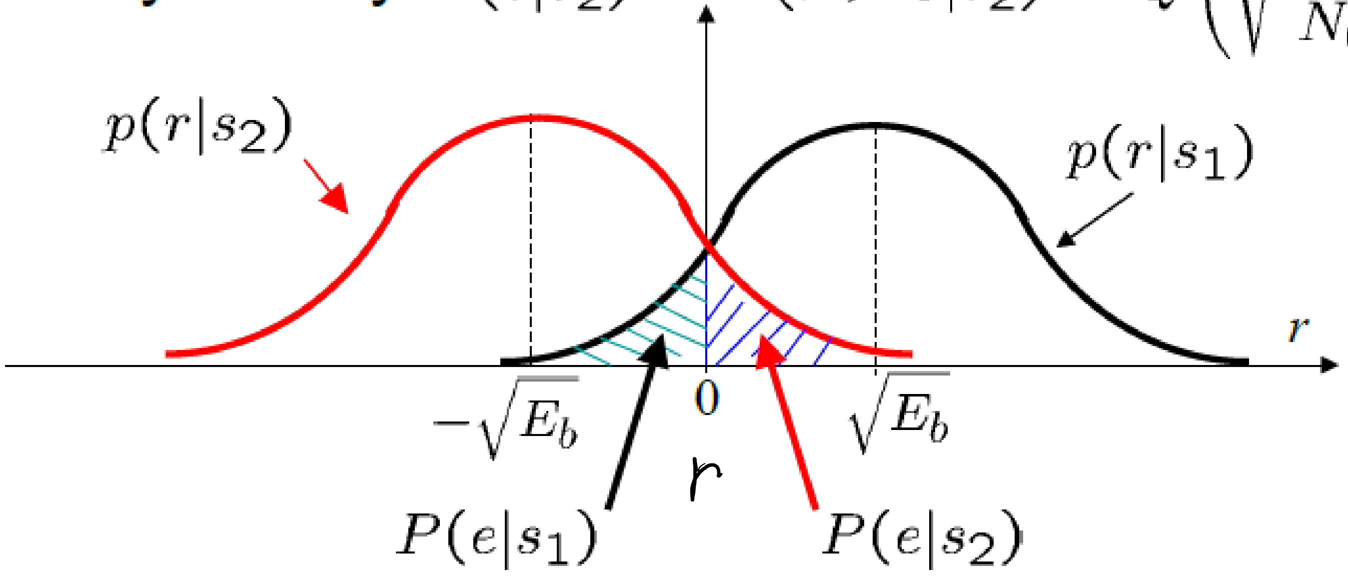


Due to symmetry $P(e|s_2) = P(r > 0|s_2) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$



$$r = \frac{N_0}{4\sqrt{E_b}} log \frac{1-p}{p}$$

Due to symmetry $P(e|s_2) = P(r > 0|s_2) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$



$r = 0$ ak

$p(r|s_1) = p(r|s_2)$

$r = r_2$ ak

$p(r|s_2) < p(r|s_1)$

$r = r_1$ ak

$p(r|s_1) < p(r|s_2)$

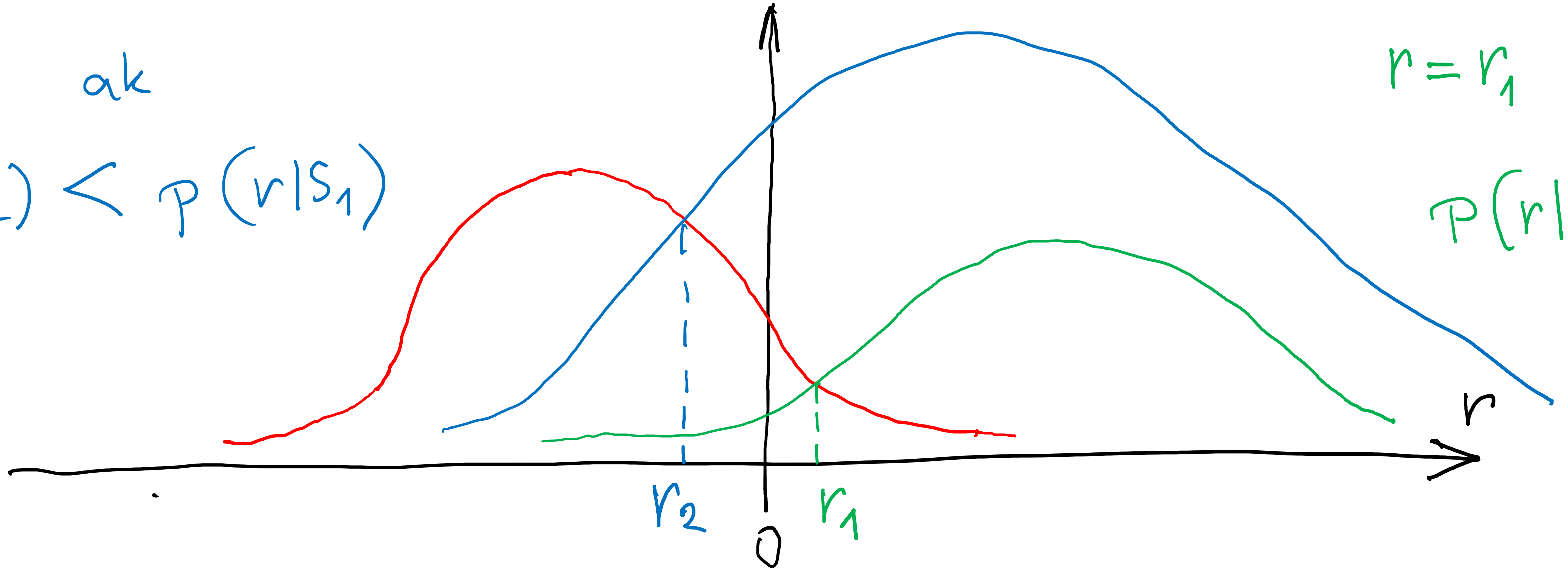


Table 1: Values of $Q(x)$ for $0 \leq x \leq 9$

x	$Q(x)$	x	$Q(x)$	x	$Q(x)$	x	$Q(x)$
0.00	0.5	2.30	0.010724	4.55	2.6823×10^{-6}	6.80	5.231×10^{-12}
0.05	0.48006	2.35	0.0093867	4.60	2.1125×10^{-6}	6.85	3.6925×10^{-12}
0.10	0.46017	2.40	0.0081975	4.65	1.6597×10^{-6}	6.90	2.6001×10^{-12}
0.15	0.44038	2.45	0.0071428	4.70	1.3008×10^{-6}	6.95	1.8264×10^{-12}
0.20	0.42074	2.50	0.0062097	4.75	1.0171×10^{-6}	7.00	1.2798×10^{-12}
0.25	0.40129	2.55	0.0053861	4.80	7.9333×10^{-7}	7.05	8.9459×10^{-13}
0.30	0.38209	2.60	0.0046612	4.85	6.1731×10^{-7}	7.10	6.2378×10^{-13}
0.35	0.36317	2.65	0.0040246	4.90	4.7918×10^{-7}	7.15	4.3389×10^{-13}
0.40	0.34458	2.70	0.003467	4.95	3.7107×10^{-7}	7.20	3.0106×10^{-13}
0.45	0.32636	2.75	0.0029798	5.00	2.8665×10^{-7}	7.25	2.0839×10^{-13}
0.50	0.30854	2.80	0.0025551	5.05	2.2091×10^{-7}	7.30	1.4388×10^{-13}
0.55	0.29116	2.85	0.002186	5.10	1.6983×10^{-7}	7.35	9.9103×10^{-14}
0.60	0.27425	2.90	0.0018658	5.15	1.3024×10^{-7}	7.40	6.8092×10^{-14}
0.65	0.25785	2.95	0.0015889	5.20	9.9644×10^{-8}	7.45	4.667×10^{-14}
0.70	0.24196	3.00	0.0013499	5.25	7.605×10^{-8}	7.50	3.1909×10^{-14}
0.75	0.22663	3.05	0.0011442	5.30	5.7901×10^{-8}	7.55	2.1763×10^{-14}
0.80	0.21186	3.10	0.0009676	5.35	4.3977×10^{-8}	7.60	1.4807×10^{-14}
0.85	0.19766	3.15	0.00081635	5.40	3.332×10^{-8}	7.65	1.0049×10^{-14}
0.90	0.18406	3.20	0.00068714	5.45	2.5185×10^{-8}	7.70	6.8033×10^{-15}
0.95	0.17106	3.25	0.00057703	5.50	1.899×10^{-8}	7.75	4.5946×10^{-15}
1.00	0.15866	3.30	0.00048342	5.55	1.4283×10^{-8}	7.80	3.0954×10^{-15}
1.05	0.14686	3.35	0.00040406	5.60	1.0718×10^{-8}	7.85	2.0802×10^{-15}
1.10	0.13567	3.40	0.00033693	5.65	8.0224×10^{-9}	7.90	1.3945×10^{-15}
1.15	0.12507	3.45	0.00028029	5.70	5.9904×10^{-9}	7.95	9.3256×10^{-16}
1.20	0.11507	3.50	0.00023263	5.75	4.4622×10^{-9}	8.00	6.221×10^{-16}
1.25	0.10565	3.55	0.00019262	5.80	3.3157×10^{-9}	8.05	4.1397×10^{-16}
1.30	0.0968	3.60	0.00015911	5.85	2.4579×10^{-9}	8.10	2.748×10^{-16}
1.35	0.088508	3.65	0.00013112	5.90	1.8175×10^{-9}	8.15	1.8196×10^{-16}
1.40	0.080757	3.70	0.0001078	5.95	1.3407×10^{-9}	8.20	1.2019×10^{-16}
1.45	0.073529	3.75	8.8417×10^{-5}	6.00	9.8659×10^{-10}	8.25	7.9197×10^{-17}
1.50	0.066807	3.80	7.2348×10^{-5}	6.05	7.2423×10^{-10}	8.30	5.2056×10^{-17}
1.55	0.060571	3.85	5.9059×10^{-5}	6.10	5.3034×10^{-10}	8.35	3.4131×10^{-17}
1.60	0.054799	3.90	4.8096×10^{-5}	6.15	3.8741×10^{-10}	8.40	2.2324×10^{-17}
1.65	0.049471	3.95	3.9076×10^{-5}	6.20	2.8232×10^{-10}	8.45	1.4565×10^{-17}
1.70	0.044565	4.00	3.1671×10^{-5}	6.25	2.0523×10^{-10}	8.50	9.4795×10^{-18}
1.75	0.040059	4.05	2.5609×10^{-5}	6.30	1.4882×10^{-10}	8.55	6.1544×10^{-18}
1.80	0.03593	4.10	2.0658×10^{-5}	6.35	1.0766×10^{-10}	8.60	3.9858×10^{-18}
1.85	0.032157	4.15	1.6624×10^{-5}	6.40	7.7688×10^{-11}	8.65	2.575×10^{-18}
1.90	0.028717	4.20	1.3346×10^{-5}	6.45	5.5925×10^{-11}	8.70	1.6594×10^{-18}
1.95	0.025588	4.25	1.0689×10^{-5}	6.50	4.016×10^{-11}	8.75	1.0668×10^{-18}
2.00	0.02275	4.30	8.5399×10^{-6}	6.55	2.8769×10^{-11}	8.80	6.8408×10^{-19}
2.05	0.020182	4.35	6.8069×10^{-6}	6.60	2.0558×10^{-11}	8.85	4.376×10^{-19}
2.10	0.017864	4.40	5.4125×10^{-6}	6.65	1.4655×10^{-11}	8.90	2.7923×10^{-19}
2.15	0.015778	4.45	4.2935×10^{-6}	6.70	1.0421×10^{-11}	8.95	1.7774×10^{-19}
2.20	0.013903	4.50	3.3977×10^{-6}	6.75	7.3923×10^{-12}	9.00	1.1286×10^{-19}
2.25	0.012224						

Error Function Table

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

	Hundredths digit of x									
x	0	1	2	3	4	5	6	7	8	9
0.0	0.00000	0.01128	0.02256	0.03384	0.04511	0.05637	0.06762	0.07886	0.09008	0.10128
0.1	0.11246	0.12362	0.13476	0.14587	0.15695	0.16800	0.17901	0.18999	0.20094	0.21184
0.2	0.22270	0.23352	0.24430	0.25502	0.26570	0.27633	0.28690	0.29742	0.30788	0.31828
0.3	0.32863	0.33891	0.34913	0.35928	0.36936	0.37938	0.38933	0.39921	0.40901	0.41874
0.4	0.42839	0.43797	0.44747	0.45689	0.46623	0.47548	0.48466	0.49375	0.50275	0.51167
0.5	0.52050	0.52924	0.53790	0.54646	0.55494	0.56332	0.57162	0.57982	0.58792	0.59594
0.6	0.60386	0.61168	0.61941	0.62705	0.63459	0.64203	0.64938	0.65663	0.66378	0.67084
0.7	0.67780	0.68467	0.69143	0.69810	0.70468	0.71116	0.71754	0.72382	0.73001	0.73610
0.8	0.74210	0.74800	0.75381	0.75952	0.76514	0.77067	0.77610	0.78144	0.78669	0.79184
0.9	0.79691	0.80188	0.80677	0.81156	0.81627	0.82089	0.82542	0.82987	0.83423	0.83851
1.0	0.84270	0.84681	0.85084	0.85478	0.85865	0.86244	0.86614	0.86977	0.87333	0.87680
1.1	0.88021	0.88353	0.88679	0.88997	0.89308	0.89612	0.89910	0.90200	0.90484	0.90761
1.2	0.91031	0.91296	0.91553	0.91805	0.92051	0.92290	0.92524	0.92751	0.92973	0.93190
1.3	0.93401	0.93606	0.93807	0.94002	0.94191	0.94376	0.94556	0.94731	0.94902	0.95067
1.4	0.95229	0.95385	0.95538	0.95686	0.95830	0.95970	0.96105	0.96237	0.96365	0.96490
1.5	0.96611	0.96728	0.96841	0.96952	0.97059	0.97162	0.97263	0.97360	0.97455	0.97546
1.6	0.97635	0.97721	0.97804	0.97884	0.97962	0.98038	0.98110	0.98181	0.98249	0.98315
1.7	0.98379	0.98441	0.98500	0.98558	0.98613	0.98667	0.98719	0.98769	0.98817	0.98864
1.8	0.98909	0.98952	0.98994	0.99035	0.99074	0.99111	0.99147	0.99182	0.99216	0.99248
1.9	0.99279	0.99309	0.99338	0.99366	0.99392	0.99418	0.99443	0.99466	0.99489	0.99511
2.0	0.99532	0.99552	0.99572	0.99591	0.99609	0.99626	0.99642	0.99658	0.99673	0.99688
2.1	0.99702	0.99715	0.99728	0.99741	0.99753	0.99764	0.99775	0.99785	0.99795	0.99805
2.2	0.99814	0.99822	0.99831	0.99839	0.99846	0.99854	0.99861	0.99867	0.99874	0.99880
2.3	0.99886	0.99891	0.99897	0.99902	0.99906	0.99911	0.99915	0.99920	0.99924	0.99928
2.4	0.99931	0.99935	0.99938	0.99941	0.99944	0.99947	0.99950	0.99952	0.99955	0.99957
2.5	0.99959	0.99961	0.99963	0.99965	0.99967	0.99969	0.99971	0.99972	0.99974	0.99975
2.6	0.99976	0.99978	0.99979	0.99980	0.99981	0.99982	0.99983	0.99984	0.99985	0.99986
2.7	0.99987	0.99987	0.99988	0.99989	0.99989	0.99990	0.99991	0.99991	0.99992	0.99992
2.8	0.99992	0.99993	0.99993	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995	0.99996
2.9	0.99996	0.99996	0.99996	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998
3.0	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99999	0.99999	0.99999
3.1	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999
3.2	0.99999	0.99999	0.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Priklad

Vysielajme 3 bitovu kombinaciu nul a jedniciek (1 symbol sa rovna 3 bitom), napr. 000, 100, 110, atd.
Pravdepodobnost vyskytu jednicky pri prenose je p=1/2.
Aka je pravdepodobnost, ze v trojici bitov sa vyskytnu dve jednicku?

Riesenie

$$P(Y=2)=\binom{3}{2}p^2(1-p)^{3-2}=\frac{3!}{2!\,1!}p^2(1-p)=\frac{3.2.1}{2.1}\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)=\frac{3}{8}$$

Prakticky dokaz

000	p=1/8			
001	p=1/8			
010	p=1/8			
011	p=1/8	011	p=1/8	
100	p=1/8			P=3/8
101	p=1/8	101	p=1/8	
110	p=1/8	110	p=1/8	
111	p=1/8			