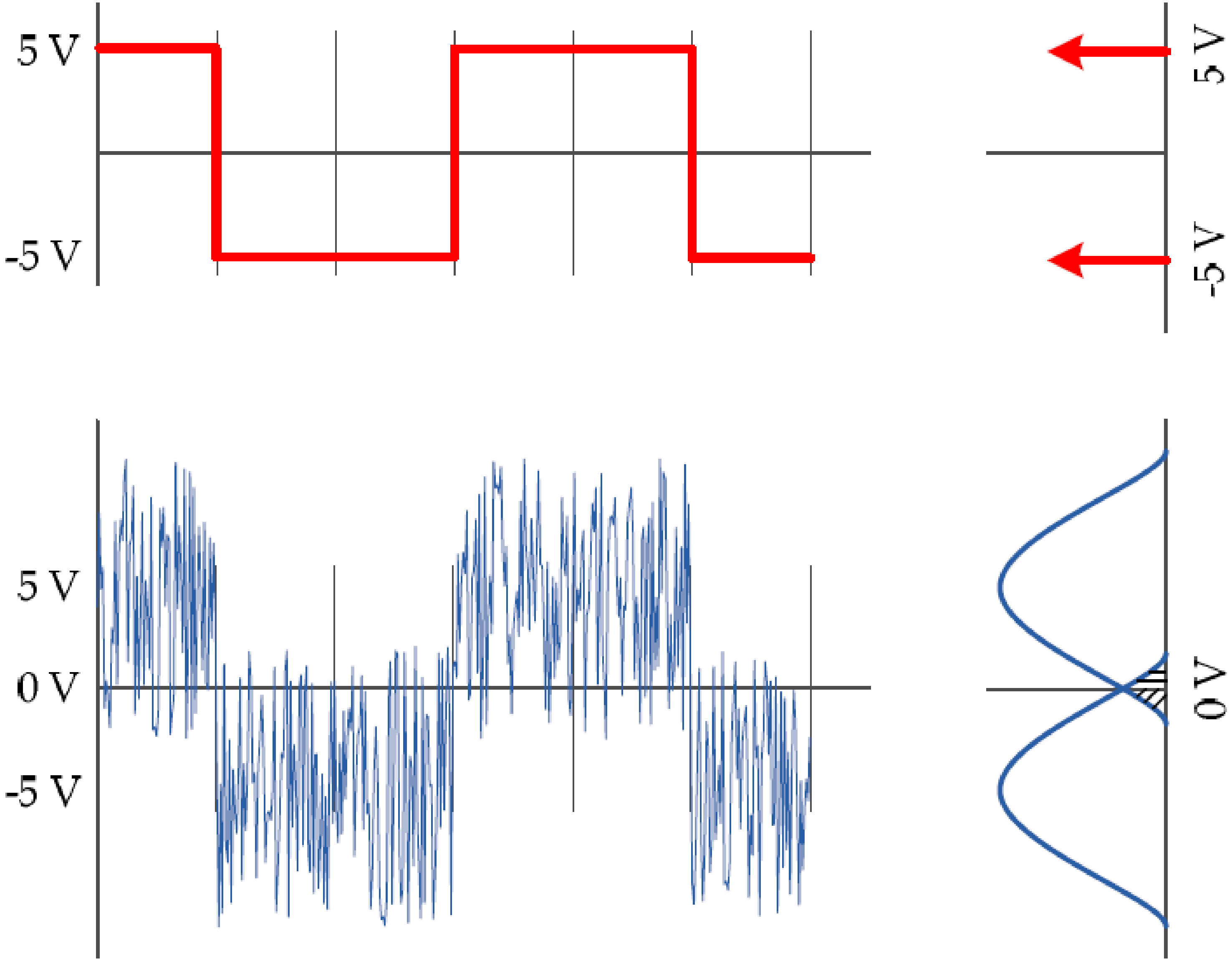
Threshold at the receiver

In which case is zero

In which case is non zero


5 V
-5 V
5 V
-5 V
5 V
0 V
-5 V
0 V

$$r = \frac{N_0}{4\sqrt{E_b}} \ln \frac{1-p}{p}$$

Problem 2

Consider the case of binary PAM signals in which the two possible signal points are $s_1 = -s_2 = \sqrt{E_b}$, where E_b is the energy per bit. The prior probabilities are $P(s_1) = p$ and $P(s_2) = 1 - p$. Let us determine the metrics for the optimum map detector when the transmitted signal is corrupted with AWGN.

Solution

The received signal vector (one-dimensional) for binary PAM is

$$r = \pm\sqrt{E_b} + y_n(t) \tag{6.62}$$

where $y_n(t)$ is a zero-mean Gaussian random variable with variance $\sigma_n^2 = \frac{1}{2}N_0$. Consequently, the conditional PDFs $p(r|s_m)$ for the two signals are

$$p(r \mid s_1) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left[-\frac{(r-\sqrt{E_b})^2}{2\sigma_n^2}\right] \tag{6.63}$$

$$p(r \mid s_2) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left[-\frac{(r+\sqrt{E_b})^2}{2\sigma_n^2}\right] \tag{6.64}$$

signals are not equally probable, the optimum MAP detector bases its decision on the probabilities P(sm|r), m=1,2...,M, given by Equation 6.17 or, equivalently, on the metrics

$$PM(r, s_m) = p(r|s_m)P(s_m)$$

Then the metrics $PM(r, s_1)$ and $PM(r, s_2)$ are

$$PM(r, s_1) = pp(r \mid s_1) = \frac{p}{\sqrt{2\pi}\sigma_n} \exp\left[-\frac{(r-\sqrt{E_b})^2}{2\sigma_n^2}\right] \tag{6.65}$$

$$PM(r, s_2) = pp(r \mid s_2) = \frac{1-p}{\sqrt{2\pi}\sigma_n} \exp\left[-\frac{(r+\sqrt{E_b})^2}{2\sigma_n^2}\right] \tag{6.66}$$

If $PM(r, s_1) > PM(r, s_2)$, we select s_1 as the transmitted signal; otherwise, we select s_2. This decision rule may be expressed as

$$\frac{PM(r, s_1)}{PM(r, s_2)} \underset{s_2}{\overset{s_1}{\gtrless}} 1 \tag{6.67}$$

But

$$\frac{PM(r, s_1)}{PM(r, s_2)} = \frac{p}{1-p} \exp\left[\frac{\left(r+\sqrt{E_b}\right)^2 - \left(r-\sqrt{E_b}\right)^2}{2\sigma_n^2}\right] \tag{6.68}$$

So that Equation 6.30 may be expressed as

$$\left[\frac{\left(r+\sqrt{E_b}\right)^2 - \left(r-\sqrt{E_b}\right)^2}{2\sigma_n^2}\right] \underset{s_2}{\overset{s_1}{\gtrless}} \ln\frac{1-p}{p} \tag{6.69}$$

or equivalently,

$$r\sqrt{E_b} \underset{s_2}{\overset{s_1}{\gtrless}} \frac{1}{2}\sigma_n^2 \ln\frac{1-p}{p} = \frac{1}{4} N_0 \ln\frac{1-p}{p} \tag{6.70}$$

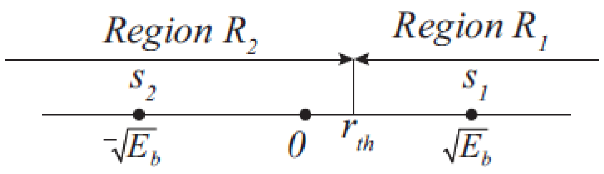


Figure 6.23 Signal space representation illustrating the operation of the optimum detector for binary PAM modulation.

This is the final form for the optimum detector. It computes the correlation metric $C(r, s_1) = r\sqrt{E_b}$ and compares it with threshold $\frac{1}{4}N_0 \ln\frac{1-p}{p}$. Figure 6.23 illustrates the two signal points s_1 and s_2. The threshold, denoted by hτ, divides the real line into two regions, say R_1 and R_2, where R_1 consists of the set of points that are greater than τ_h, and R_2 consists of the set of points that are less than τ_h. If $r\sqrt{E_b} > \tau_h$, the decision is made that s_1 was transmitted, and if $\sqrt{E_b} < \tau_h$, the decision is made that s_2 was transmitted. The threshold τ_h depends on N_0 and p. If $p = \frac{1}{2}$, $\tau_h = 0$. If p>1/2, the signal point s_1 is more probable, and, hence, $\tau_h < 0$. In this case, the region R_1 is larger than R_2, so that s_1 is more likely to be selected than s_2. If $p < \frac{1}{2}$, the opposite is the case. Thus, the average probability of error is minimized.

It is interesting to note that in the case of unequal prior probabilities, it is necessary to know not only the values of the prior probabilities but also the value of the power spectral density N_0, or, equivalently, the noise-to-signal ration $\frac{N_0}{E_b}$, in order to compute the threshold. When $p = \frac{1}{2}$, the threshold is zero, and the knowledge of N_0 is not required by the detector.