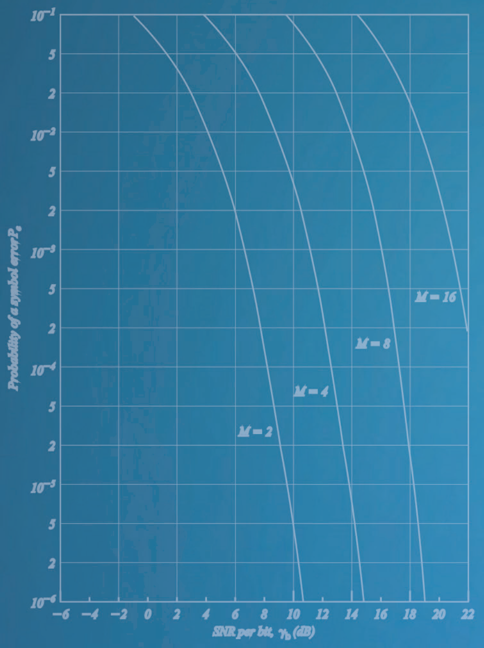

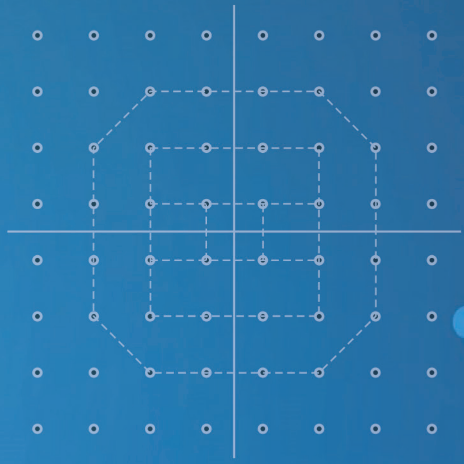

Theory of Telecommunications Networks

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CONTENTS

PREFACE

Providing the theory of digital communication systems, this textbook prepares senior undergraduate and graduate students for the engineering practices required in the real word.

With this textbook, students can understand how digital communication systems operate in practice, learn how to design subsystems, and evaluate end-to-end performance.

The book contains many examples to help students achieve an understanding of the subject. The problems are at the end of the each chapter follow closely the order of the sections.

The entire book is suitable for one semester course in digital communication.

All materials for teaching texts were drawn from sources listed in References.

7 PERFORMANCE ANALYSIS OF DIGITAL MODULATIONS

7.1 GOALS OF THE COMMUNICATIONS SYSTEM DESIGNER

System trade-offs are fundamental to all digital communication designs. The goals of the designer may include any of the following:

1. *to maximize transmission bit rate R;*
2. *to minimize probability of bit error* P_B *;*
3. *to minimize required power, or equivalently, to minimize required bit energy to noise power spectral density* $\frac{E_b}{N_0}$ *;*
4. *to minimize required system bandwidth B;*
5. *to maximize system utilization, that is, to provide reliable service for a maximum number of users with minimum delay and with maximum resistance to interference;*
6. *to minimize system complexity, computational load, and system cost. A system designer may seek to achieve all these goals simultaneously.*

However, goals 1 and 2 are clearly in conflict with goals 3 and 4; they call for simultaneously maximizing *R*, while minimizing P_B , $\frac{E_b}{N_0}$, and *B*. There are several constraints and theoretical limitations that necessitate the trading off of any one system requirement with each of the others:

- The Nyquist theoretical minimum bandwidth requirement
- The Shannon-Hartley capacity theorem (and the Shannon limit)
- Government regulations (e.g., frequency allocations)
- Technological limitations (e.g., state-of-the-art components)
- Other system requirements (e.g., satellite orbits)

Some of the realizable modulation and coding trade-offs can best be viewed as a change in operating point on one of two performance planes. These planes will be referred to as the error probability plane and the bandwidth efficiency plane, and they are described in the following sections.

7.2 ERROR PROBABILITY PLANE

Figure 7.1 illustrates the family of P_B versus $\frac{E_b}{N_0}$ curves for the coherent detection of orthogonal signaling (Figure 7.1 a)) and multiple phase signaling (Figure 7.1 b)). The modulator uses one of its $M = 2^k$ waveforms to represent each k-bit sequence, where M is the size of the symbol set. Figure 7.1 a) illustrates the potential bit error improvement with orthogonal signaling as k (or M) is increased. For orthogonal signal sets, such as orthogonal frequency shift keying (FSK) modulation, increasing the size of the symbol set can provide an improvement in P_B, or a reduction in the $\frac{E_b}{N_0}$ required, at the cost of increased bandwidth. Figure 7.1 b) illustrates potential bit error degradation with nonorthogonal signaling as k (or M) increases. For nonorthogonal signal sets, such as multiple phase shift keying (MPSK) modulation, increasing the size of the symbol set can reduce the bandwidth requirement, but at the cost of a degraded P_B, or an increased $\frac{E_b}{N_0}$ requirement. We shall refer to these families of curves (Figure 7.1 a) or b) as e*rror probability performance curves*, and to the plane on which they are plotted as an *error probability plane*. Such a plane describes the locus of operating points available for a particular type of modulation and coding. For a given system information rate, each curve in the plane can be associated with a different fixed minimum required bandwidth; therefore, the set of curves can be termed *equibandwidth curves*. As the curves move in the direction of the ordinate, the required transmission bandwidth increases; as the curves move in the opposite direction, the required bandwidth decreases. Once a modulation and coding scheme and an available $\frac{E_b}{N_0}$ are determined, a particular point in the error probability plane characterizes system operation.

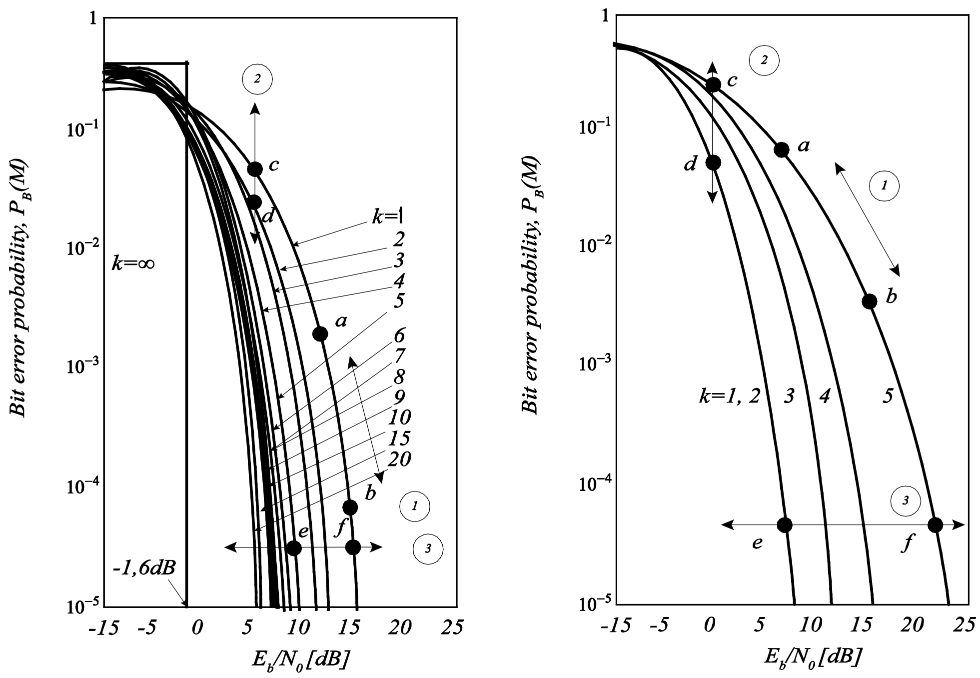


Figure 7.1 Bit error probability versus $\frac{E_b}{N_0}$ for coherently detected M-ary signaling: a) Orthogonal signaling, b) Multiple phase signaling.

Possible trade-offs can be viewed as changes in the operating point on one of the curves or as changes in the operating point from one curve to another curve of the family. These trade-offs are seen in Figure 7.1 a) and b) as changes in the system operating point in the direction shown by the arrows. Movement of the operating point along line 1, between points a and b, can be viewed as trading off between P_B and $\frac{E_b}{N_0}$ performance (with W fixed). Similarly, movement along line 2, between points c and d, is seen as trading P_B versus B (with $\frac{E_b}{N_0}$ fixed). Finally, movement along line 3, between points e and f, illustrates trading W versus $\frac{E_b}{N_0}$ (with P_B fixed). Movement along line 1 is effected by increasing or decreasing the available $\frac{E_b}{N_0}$. This can be achieved, for example, by increasing transmitter power, which means that the trade-off might be accomplished simply by "turning a knob," even after the system is configured. However, the other trade-offs (movement along line 2 or line 3) involve some changes in the system modulation or coding scheme, and therefore need to be accomplished during the system design phase. The advent of software radios will even allow changes to a system's modulation and coding by programmable means.

7.3 NYQUIST MINIMUM BANDWIDTH

Every realizable system having some nonideal filtering will suffer from intersymbol interference (ISI)- the tail of one pulse spilling over into adjacent symbol intervals so as to interfere with correct detection. Nyquist showed that the theoretical minimum bandwidth (Nyquist bandwidth) needed for the baseband transmission of R_s symbols per second without ISI is $\frac{R_s}{2}$ hertz. This is a basic theoretical constraint, limiting the designer's goal to expend as little bandwidth as possible. In practice, the Nyquist minimum bandwidth is expanded by about 10% to 40%, because of the constraints of real filters. Thus, typical baseband digital communication throughput is reduced from the ideal $2\ symbols/s/Hz$ to the range of about $1{,}8\ to\ 1{,}4\ symbols/s/Hz$. From its set of M symbols, the modulation or coding system assigns to each symbol a k-bit meaning, where $M = 2^k$. Thus, the number of bits per symbol can be expressed as $k = \log_2 M$, and the data rate or bit rate R must be k times faster than the symbol rate R_s, as expressed by the basic relationship

$$\mathrm{R} = \mathrm{k}R_s\ or\ R_s = \frac{R}{k} = \frac{R}{\log_2 M} \tag{7.1}$$

For signaling at a fixed symbol rate, Equation 7.1 shows that, as k is increased, the data rate R is increased. In the case of MPSK, increasing k, thereby results in an increased bandwidth efficiency R/B measured in bits/s/Hz. For example, movement along line 3, from point e to point f in Figure 7.1 b), represents trading $\frac{E_b}{N_0}$ for a reduced bandwidth requirement. In other words, with the same system bandwidth, one can transmit MPSK signals at an increased date rate and hence at an increased $\frac{R}{W}$.

7.4 SHANNON-HARTLEY CAPACITY THEOREM

Shannon showed that the system capacity *C* of a channel perturbed by additive white Gaussian noise (AWGN) is a function of the average received signal power *S*, the average noise power *N*, and the bandwidth B. The capacity relationship (Shannon-Hartley theorem) can be stated as

$$\mathrm{C} = \mathrm{B}\log_2\left(1 + \frac{S}{N}\right) \tag{7.2}$$

When B is in hertz and the logarithm is taken to the base 2, as shown, the capacity is given in $bits/s$. It is theoretically possible to transmit information over such a channel at any rate R, where $R \leq C$, with an arbitrarily small error probability by using a sufficiently complicated coding scheme. For an information rate $R > C,$ it is not possible to find a code that can achieve an arbitrarily small error probability. Shannon's work showed that the values of S, N, and B set a limit on transmission rate, not on error probability. Shannon used Equation 7.2 to graphically exhibit a bound for the achievable performance of practical systems. This plot, shown in Figure 7.2, gives the normalized channel capacity $\frac{C}{B}$ in bits/s/Hz as a function of the channel signal-to-noise ratio (SNR). A related plot. shown in Figure 7.3, indicates the normalized channel bandwidth B/C in Hz/bits/s as a function of SNR in the channel. is sometimes used to illustrate the power-bandwidth tradeoff inherent in the ideal channel. However, it is not a pure trade-off because the detected noise power is proportional to bandwidth:

$$\mathrm{N} = \mathrm{N}_0 B \tag{7.3}$$

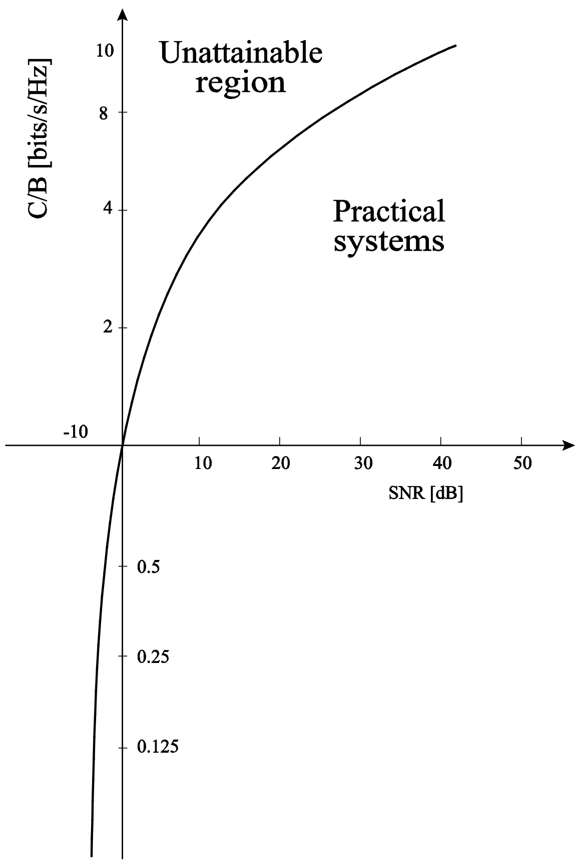


Figure 7.2 Normalized channel capacity versus channel SNR.

Substituting Equation 7. 3 into Equation 7.2 and rearranging terms yields

$$\frac{C}{B} = \log_2\left(1 + \frac{S}{N_0 B}\right) \tag{7.4}$$

For the case where transmission bit rate is equal to channel capacity, R = C, we can use the identity to write

$$\frac{S}{N_0 C} = \frac{E_b}{N_0} \tag{7.5}$$

Hence, we can modify Equation 7.4 as follows:

$$\frac{C}{B} = \log_2\left[1 + \frac{E_b}{N_0}\left(\frac{C}{B}\right)\right] \tag{7.6}$$

$$2^{\frac{C}{B}} = 1 + \frac{E_b}{N_0}\left(\frac{C}{B}\right) \tag{7.7}$$

$$\frac{E_b}{N_0} = \frac{B}{C}\left(2^{\frac{C}{B}} - 1\right) \tag{7.8}$$

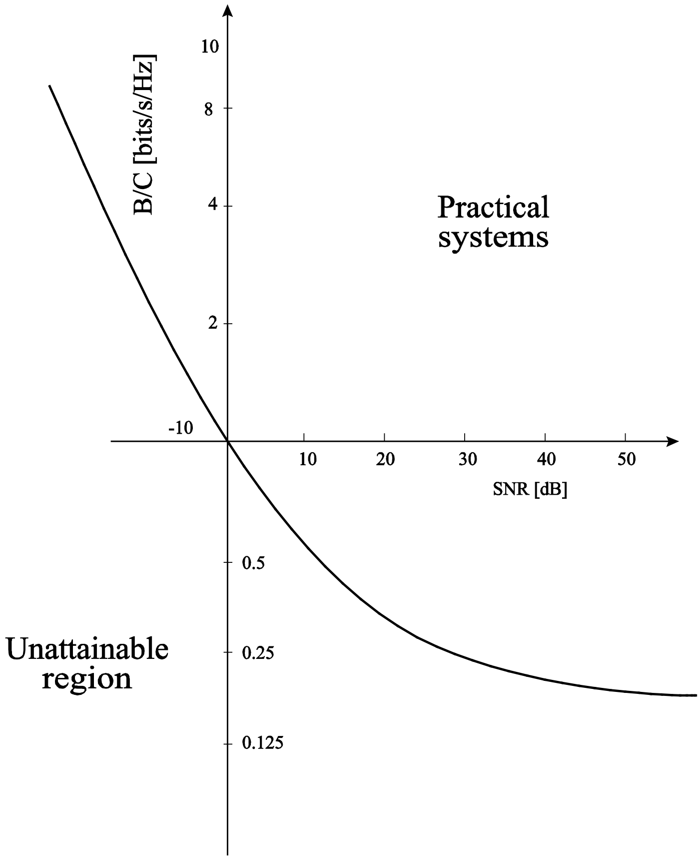


Figure 7.3 Normalized channel bandwidth versus channel SNR.

Figure 7.4 is a plot of $\frac{B}{C}$ versus $\frac{E_b}{N_0}$ in accordance with Equation 7.8. The asymptotic behavior of this curve as $\frac{C}{B} \to 0$ or (or $\frac{B}{C} \to \infty$) is discussed in the next section.

7.4.1 Shannon Limit

There exists a limiting value of $\frac{E_b}{N_0}$ below, which there can be no error-free communication at any information rate. Using the identity

$$\lim_{x \to 0}(1 + x)^{\frac{1}{x}} = e$$

We can calculate the limiting value of $\frac{E_b}{N_0}$ as follows: Let

$$x = \frac{E_b}{N_0}\left(\frac{C}{B}\right) \tag{7.5}$$

Then, from Equation 7.6,

$$\frac{C}{B} = x \log_2(1 + x)^{\frac{1}{x}} \tag{7.6}$$

And

$$1=\frac{E_b}{N_0}\log_2(1+x)^{\frac{1}{x}} \quad (7.7)$$

In the limit, as $\frac{C}{B} \rightarrow 0$, we get

$$\frac{E_b}{N_0}=\frac{1}{\log_2 e}=0{,}693 \quad (7.8)$$

Or in decibels

$$\frac{E_b}{N_0}=-1{,}6\text{dB} \quad (7.9)$$

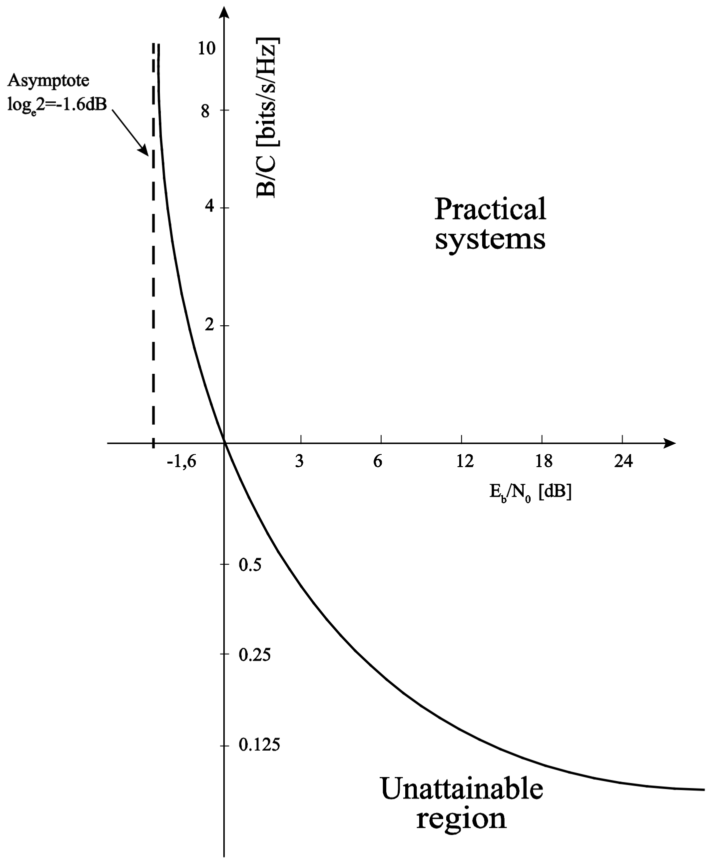


Figure 7.4 Normalized channel bandwidth versus channel $\frac{E_b}{N_0}$.

This value of $\frac{E_b}{N_0}$ is called the *Shannon limit*. On Figure 7.1 a) the Shannon limit is the P_B versus $\frac{E_b}{N_0}$ curve corresponding to $k \rightarrow \infty$. The curve is discontinuous, going from a value of $P_B=\frac{1}{2}$ to $P_B=0$ at $\frac{E_b}{N_0}=-1{,}6\ dB$. It is not possible in practice to reach the Shannon limit, because as k increases without bound, the bandwidth requirement and the implementation complexity increases without bound. Shannon's work provided a theoretical proof for the existence of codes that could improve the P_B

performance, or reduce the $\frac{E_b}{N_0}$ required, from the levels of the uncoded binary modulation schemes to levels approaching the limiting curve. For a bit error probability of 10^{-5} , binary phase-shift-keying (BPSK) modulation requires an $\frac{E_b}{N_0}$ of 9.6 dB (the optimum uncoded binary modulation). Therefore, for this case, Shannon's work promised the existence of a theoretical performance improvement of 11.2 dB over the performance of optimum uncoded binary modulation, through the use of coding techniques. Today, most of that promised improvement (as much as 10 dB) is realizable with turbo codes. Optimum system design can best be described as a search for rational compromises or trade-offs among the various constraints and conflicting goals. The modulation and coding trade-off, that is, the selection of modulation and coding techniques to make the best use of transmitter power and channel bandwidth, is important, since there are strong incentives to reduce the cost of generating power and to conserve the radio spectrum.

7.5 BANDWIDTH-EFFICIENCY PLANE

Using Equation 7.6, we can plot normalized channel bandwidth $\frac{B}{C}$ in $Hz/bits/s$ versus $\frac{E_b}{N_0}$, as shown in Figure 7.4. Here, with the abscissa taken as $\frac{E_b}{N_0}$, we see the *true power-bandwidth trade-off* at work. It can be shown [5] that well designed systems tend to operate near the "knee" of this power-bandwidth tradeoff curve for the ideal (R = C) channel. Actual systems are frequently within 10 dB or less of the performance of the ideal. The existence of the knee means that systems seeking to reduce the channel bandwidth they occupy or to reduce the signal power they require must make an increasingly unfavorable exchange in the other parameter. For example, from Figure 7.4, an ideal system operating at an $\frac{E_b}{N_0}$ of 1.8 dB and using a normalized bandwidth of 0.5 Hz/bits/s would have to increase $\frac{E_b}{N_0}$ to 20 dB to reduce the bandwidth occupancy to 0.1 Hz/bits/s. Trade-offs in the other direction are similarly inequitable.

Using Equation 7.8, we an also plot $\frac{C}{B}$ versus $\frac{E_b}{N_0}$. This relationship is shown plotted on the $\frac{R}{B}$ versus $\frac{E_b}{N_0}$ plane in Figure 7.5. We shall denote this plane as the bandwidth-efficiency plane. The ordinate $\frac{R}{B}$ is a measure of how much data can be communicated in a specified bandwidth within a given time; it therefore reflects how efficiently the bandwidth resource is utilized. The abscissa is $\frac{E_b}{N_0}$, in units of decibels. For the case in which R = C in Figure 7.5, the curve represents a boundary that separates a region characterizing practical communication systems from a region where such communication systems are not theoretically possible. Like Figure 7.2, the bandwidth-efficiency plane in Figure 7.5 sets the limiting performance that can be achieved by practical systems. Since the abscissa in Figure 7.5 is $\frac{E_b}{N_0}$ rather than SNR, Figure 7.5 is more useful for comparing digital communication modulation and coding trade-offs than is Figure 7.2. Note that Figure 7.5 illustrates bandwidth efficiency versus $\frac{E_b}{N_0}$ for single-carrier systems. For multiple-carrier systems, bandwidth efficiency is also a function of carrier spacing (which depends on the modulation type). The trade-off becomes how closely can the carriers be spaced (thereby improving bandwidth efficiency) without suffering an unacceptable amount of adjacent channel interference (ACI).

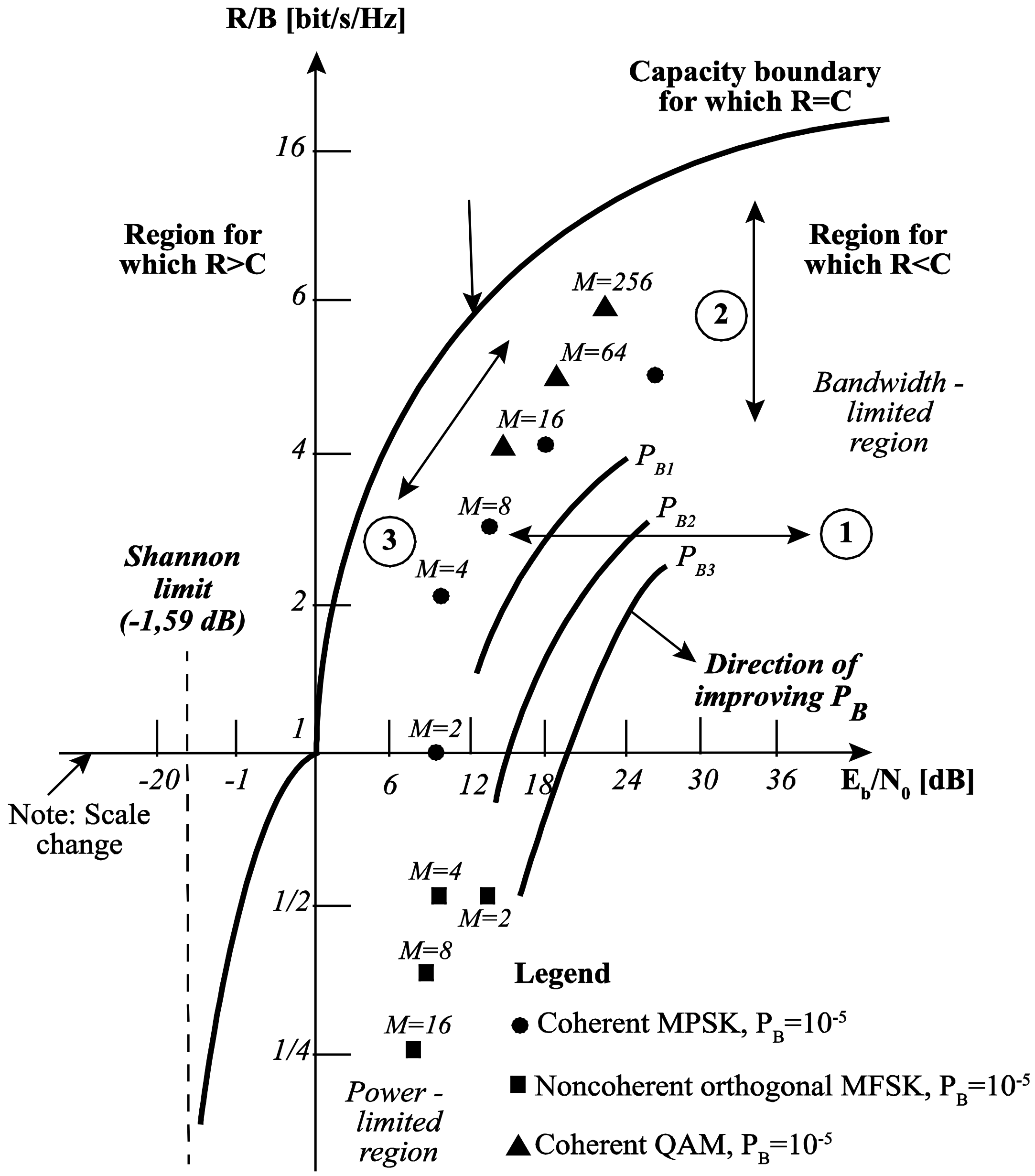


Figure 7.5 Bandwidth-efficiency plane.

7.5.1 Bandwidth Efficiency of MPSK and MFSK Modulation

On the bandwidth-efficiency plane of Figure 7.5 are plotted the operating points for coherent MPSK modulation at a bit error probability of 10^{-5}. We assume Nyquist (ideal rectangular) filtering at baseband, so that the minimum double-sideband (DSB) bandwidth at an intermediate frequency (IF) is $B_{IF} = \frac{1}{T}$, where T is the symbol duration. Thus using (Equation 7.1) the bandwidth efficiency is $\frac{R}{B} = \log_2 M$, where M is the symbol set size. For realistic channels and waveforms, the performance must be reduced to account for the bandwidth increase required to implement realizable filters. Notice that for MPSK modulation. $\frac{R}{B}$ increases with increasing M. Notice also that the location of the MPSK points indicates that BPSK (M = 2) and quaternary PSK or QPSK (M = 4) require the same $\frac{E_b}{N_0}$. That is, for the same value of $\frac{E_b}{N_0}$, QPSK has a bandwidth efficiency of 2 bits/s/Hz, compared to 1 bit/s/Hz for

BPSK. This unique features stems from the fact that QPSK is effectively a composite of two BPSK signals transmitted on orthogonal components of the carrier.

Also plotted on the bandwidth-efficiency plane of Figure 7.5 are the operating points for noncoherent orthogonal MFSK modulation, at a bit error probability of 10^{-5}. We assume that the IF transmission bandwidth is $B_{IF} = \frac{M}{T}$, and thus using Equation 7.1, the bandwidth efficiency is $\frac{R}{B} = \frac{\log_2 M}{M}$. Notice that for MFSK modulation, $\frac{R}{B}$ decreases with increasing M. Notice also that the position of the MFSK points indicates that BFSK (M = 2) and quaternary FSK (M = 4) have the same bandwidth efficiency, even though the former requires greater $\frac{E_b}{N_0}$ for the same error probability. The bandwidth efficiency varies with the modulation index (tone spacing in hertz divided by bit rate). Under the assumption that an equal increment of bandwidth is required for each MFSK tone the system uses, it can be seen that for M = 2, the bandwidth efficiency is l bit/s/2 Hz or $\frac{1}{2}$ and for M = 4, similarly, the $\frac{R}{B}$ is 2 bits/s/4 Hz or $\frac{1}{2}$. Thus binary and 4-ary orthogonal FSK are curiously characterized by the same value of $\frac{R}{B}$.

Operating points for coherent quadrature amplitude modulation (QAM) are also plotted in Figure 7.5. Of the modulations shown, QAM is clearly the most bandwidth efficient.

7.5.2 Analogies Between Bandwidth-Efficiency and Error-Probability Planes

The bandwidth-efficiency plane in Figure 7.5 is analogous to the error-probability plane in Figure 7.1. The Shannon limit of the Figure 7.1 plane is analogous to the capacity boundary of the Figure 7.5 plane. The curves in Figure 7.1, were referred to as equibandwidth curves. In Figure 7.5, we can analogously describe equi-error-probability curves for various modulation and coding schemes. The curves, labeled P_{B1}, P_{B2}, and P_{B3}, are hypothetical constructions for some arbitrary modulation and coding scheme; the P_{B1} curve represents the largest error probability of the three curves, and the P_{B3}curve represents the smallest. The general direction in which the curves move for improved P_B is indicated on the figure. Just as potential trade-offs among P_B , $\frac{E_b}{N_0}$, and *B* were considered for the error-probability plane, the same trade-offs can be considered on the bandwidth efficiency plane. The potential trade-offs are seen in Figure 7.5 as changes in operating point in the direction shown by the arrows. Movement of the operating point along line 1 can be viewed as trading P_B versus $\frac{E_b}{N_0}$, with $\frac{R}{B}$ fixed. Similarly, movement along line 2 is seen as trading P_B versus *B* (or $\frac{R}{B}$), with $\frac{E_b}{N_0}$ fixed.

Finally, movement along line 3 illustrates trading B (or $\frac{R}{B}$) versus $\frac{E_b}{N_0}$, with P_B fixed. In Figure 7.5, as in Figure 7.1, movement along line 1 can be effected by increasing or decreasing the available $\frac{E_b}{N_0}$. However, movement along line 2 or line 3 requires changes in the system modulation or coding scheme. The two primary communications resources are the transmitted power and the channel bandwidth. In many communication systems, one of these resources may be more precious than the other, and hence most systems can be classified as either power limited or bandwidth limited. In *power-limited systems*, coding schemes can be used to save power at the expense of bandwidth. whereas in *bandwidth-limited systems*, spectrally efficient modulation techniques can be used to save bandwidth at the expense of power.

7.6 MODULATION AND CODING TRADE-OFFS

Figure 7.6 is useful in pointing out analogies between the two performance planes, the error-probability plane of Figure 7.1 and the bandwidth-efficiency plane of Figure 7.5. Figure 7.6 a) and b) represent the same planes as Figure 7.1 and Figure 7.5, respectively. They have been redrawn as symmetrical by choosing appropriate scales. In each case, the arrows and their labels describe the general effect of moving an operating point in the direction of the arrow by means of appropriate modulation and coding techniques. The notations *G*, *C*, and *F* stand for the trade-off considerations "*Gained or achieved*," "*Cost or expended*," and "*Fixed or unchanged*," respectively. The parameters being traded are $P_B, B, \frac{R}{B}$ and P $(power\ or\ \frac{S}{N})$. Just as the movement of an operating point toward the Shannon limit in Figure 7.6 a) can achieve improved P_S or reduced required transmitter power at the cost of bandwidth, so too movement toward the capacity boundary in Figure 7.6 b) can improve bandwidth efficiency at the cost of increased required power or degraded P_S.

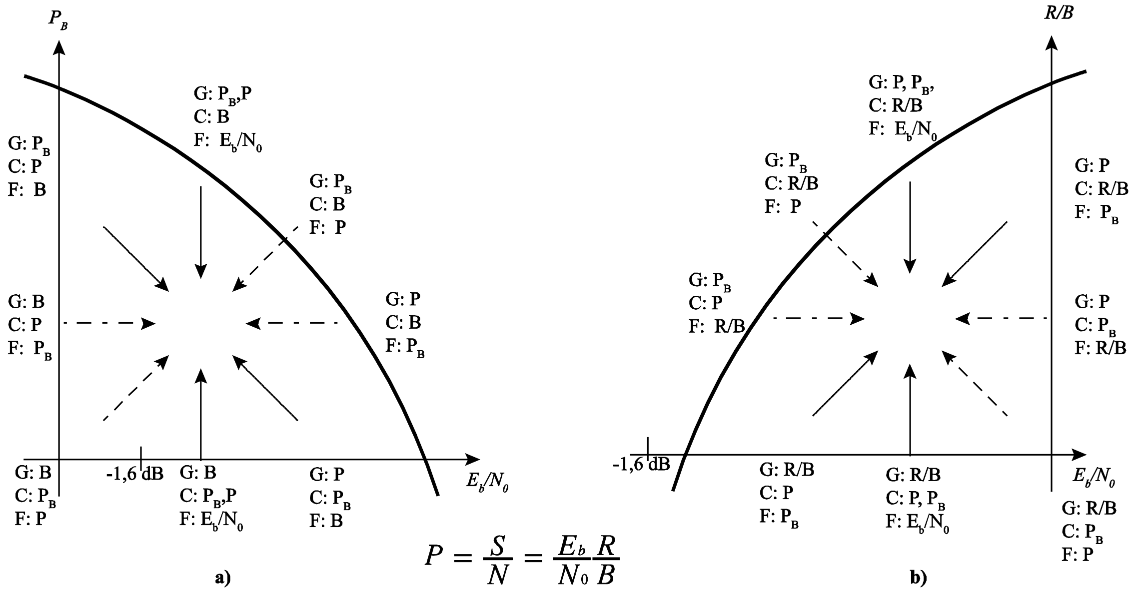


Figure 7.6 Modulation/coding trade-offs: a) Error probability plane, b) Bandwidth efficiency plane.

Most often, these trade-offs are examined with a fixed P_S (constrained by the system requirement) in mind. Therefore, the most interesting arrows are those having bit error probability (marked $F: P_B$). There are four such arrows on Figure 7.6 two on the error probability plane and two on the bandwidth-efficiency plane. Arrows marked with the same pattern indicate correspondence between the two planes. System operation can be characterized by either of these two planes. The planes represent two ways of looking at some of the key system parameters; each plane highlights slightly different aspects of the overall design problem. The error probability plane tends to be most useful with *power-limited systems*, whereas when we move from curve to curve, the bandwidth requirements are only inferred, while the bit error probability is clearly displayed. The bandwidth-efficiency plane is generally more useful for examining *bandwidth-limited systems*; here, as we move from curve to curve, the bit-error probability is only inferred, but the bandwidth requirements are explicit.

The two system trade-off planes, error probability and bandwidth efficiency, have been presented heuristically with simple examples (orthogonal and multiple phase signaling) to provide some insight into the design issues of trading-off error probability, bandwidth and power. The ideas are useful for most modulation and coding schemes, with the following caveat. For some codes or combined modulation and coding schemes, the performance curves do not move as predictably as those for the examples chosen here. The reason has to do with the error-correcting capability and bandwidth expansion features of the particular code. Examine the curves characterizing the two BCH codes, (127, 64) and (127, 36). It should be clear from their relative positions that the (127, 64) code manifests greater coding gain than the (127, 36) code. This violates our expectations since, within the same block size, the latter code has greater redundancy (requires more bandwidth expansion) than the former. Also, we consider codes that provide coding gain without any bandwidth expansion. Performance curves for such coding schemes will also behave differently from the curves of most modulation and coding schemes discussed so far.

7.7 DEFINING, DESIGNING, AND EVALUATING DIGITAL COMMUNICATION SYSTEMS

This section is intended to serve as a "*road map*" for outlining typical steps that need to be considered in meeting the bandwidth, power, and error-performance requirements of a digital communication system. The criteria for choosing modulation and coding schemes, based on whether a system is bandwidth limited or power limited, are reviewed for several system examples. We will emphasize the subtle but straightforward relationships that exist when transforming from data-bits to channel-bits to symbols to chips.

The design of any digital communication system begins with a description of the channel (received power, available bandwidth, noise statistics and other impairments, such as fading), and a definition of the system requirements (data rate and error performance). Given the channel description, we need to determine design choices that best match the channel and meet the performance requirements. An orderly set of transformations and computations has evolved to aid in characterizing a system's performance. Once this approach is understood, it can serve as the format for evaluating most communication systems. In subsequent sections, we examine three system examples, chosen to provide a representative assortment: a *bandwidth-limited uncoded system*, a *power-limited uncoded system*, and *a bandwidth-limited* and *power-limited coded system*. In this section, we deal with real-time communication systems, where the term *coded* (or *uncoded*) refers to the presence (or absence) of error-correction coding schemes involving the use of redundant bits and expanded bandwidth. Two primary communications resources are the received power and the available transmission bandwidth. In many communication systems, one of these resources may be more precious than the other, and hence most systems can be classified as either bandwidth limited or power limited. In bandwidth-limited systems, spectrally efficient modulation techniques can be used to save bandwidth at the expense of power, whereas in power-limited systems, power-efficient modulation techniques can be used to save power at the expanse of bandwidth. In both bandwidth- and power-limited systems, error-correction coding (often called channel coding) can be used to save power or to improve error performance at the expense of bandwidth. Trellis-coded modulation (TCM) schemes have been used to improve the error performance of bandwidth-limited channels without any increase in bandwidth.

7.7.1 M-ary Signaling

For signaling schemes that process k bits at a time, the signaling is called M-ary. Each symbol in an M-ary alphabet can be related to a unique sequence of k bits, where

$$M = 2^k \; or \; k = \log_2 M \tag{7.10}$$

and where M is the size of the alphabet. In the case of digital transmission, the term symbol refers to the member of the M-ary alphabet that is transmitted during each symbol duration T_s. In order to transmit the symbol, it must be mapped onto an electrical voltage or current waveform. Because the waveform represents the symbol, the terms symbol and waveform are sometimes used interchangeably. Since one of M symbols or waveforms is transmitted during each symbol duration T_s, the date rate R can be expressed as

$$R = \frac{k}{T_s} = \frac{\log_2 M}{T_s} \ [bits/s] \tag{7.11}$$

From Equation 7.11, we write that the effective duration T_b of each bit in terms of the symbol duration T_s or the symbol rate R_s is

$$T_b = \frac{1}{R} = \frac{T_s}{k} = \frac{1}{kR_s} \tag{7.12}$$

Then, using Equations 7.10 and 7.12, we can express the symbol rate R_s in terms of the bit rate R, as was presented earlier:

$$R_s = \frac{R}{\log_2 M} \tag{7.13}$$

From Equations 7.11 and 7.12, it is seen that any digital scheme that transmits $k = \log_2 M$ bits in T_s, seconds, using a bandwidth of B [Hz], operates at a bandwidth efficiency of

$$\frac{R}{B} = \frac{\log_2 M}{BT_s} = \frac{1}{BT_b} \ [bit/s/Hz] \tag{7.14}$$

where T_b is the effective time duration of each data bit.

7.7.2 Bandwidth-Limited Systems

From Equation 7.14, it can be seen that any digital communication system will become more bandwidth efficient as its BT_b product is decreased. Thus, signals with small BT_s products are often used with *bandwidth-limited* systems. For example, the Global System for Mobile (GSM) Communication uses Gaussian minimum shift keying (GMSK) modulation having a BT_b product equal to 0,3 Hz/bit/s, where B the 3-dB bandwidth of a Gaussian filter.

For uncoded bandwidth-limited systems, the objective is to maximize the transmitted information rate within the allowable bandwidth, at the expense of $\frac{E_b}{N_0}$ (while maintaining a specified value of bit-error probability P_B). On the bandwidth-efficiency plane of in Figure 7.5 are plotted the operating points for coherent M-ary PSK (MPSK) at $P_B = 10^{-5}$. We shall assume Nyquist (ideal rectangular) filtering at baseband, so that, for MPSK, the required double-sideband (DSB) bandwidth at an intermediate frequency (IF) is related to the symbol rate by

$$B = \frac{1}{T_s} = R_s \tag{7.15}$$

where T_s is the symbol duration and R_s is the symbol rate. The use of Nyquist filtering results in the minimum required transmission bandwidth that yields zero intersymbol interference; such ideal filtering gives rise to the name Nyquist minimum bandwidth. Note that the bandwidth of nonorthogonal signaling, such as MPSK or MQAM, does not depend on the density of the signaling points in the constellation but only on the speed of signaling. When a phasor is transmitted, the system cannot distinguish as to whether that signal arose from a sparse alphabet set or a dense alphabet set. It is this aspect of nonorthogonal signals that allows us to pack the signaling space densely and thus achieve improved bandwidth efficiency at the expense of power. From Equations 7.14 and 7.15, the bandwidth efficiency of MPSK modulated signals using Nyquist filtering can be expressed as

$$\frac{R}{B} = \log_2 M \;\; [bit/s/Hz] \tag{7.16}$$

The MPSK points plotted in Figure 7.5 confirm the relationship shown in Equation 7.16. Note that MPSK modulation is a bandwidth-efficient scheme. As M increases in value, $\frac{R}{B}$ also increases. From Figure 7.5, it can be verified that MPSK modulation can achieve improved bandwidth efficiency at the cost of increased $\frac{E_b}{N_0}$ • Many highly bandwidth-efficient modulation schemes have been investigated, but such schemes are beyond the scope of this book.

Two regions, the bandwidth-limited region and the power-limited region, are shown on the bandwidth-efficiency plane of Figure 7.5. Notices that the desirable trade-offs associated with each of these regions are not equitable. For the bandwidth-limited region, large $\frac{R}{B}$ is desired; however, as $\frac{E_b}{N_0}$ is increased, the capacity boundary curve flattens out and ever-increasing amounts of additional $\frac{E_b}{N_0}$ are required to achieve improvement in $\frac{R}{B}$. A similar relationship is at work in the power-limited region. Here a savings in $\frac{E_b}{N_0}$ is desired, but the capacity boundary curve is steep; to achieve a small reduction in required $\frac{E_b}{N_0}$, requires a large reduction in $\frac{R}{B}$.

7.7.3 Power-Limited Systems

For the case of power-limited systems in which power is scarce but system bandwidth is available (e.g., a space communication link), the following trade-offs, which can be seen in Figure 7.1 a), are possible: (1) improved P_B at the expense of bandwidth for a fixed $\frac{E_b}{N_0}$; or (2) reduction in $\frac{E_b}{N_0}$ at the expense of bandwidth for a fixed P_B. A "natural" modulation choice for a power-limited system is M-ary FSK (MFSK). Plotted on Figure 7.5 are the operating points for noncoherent orthogonal MFSK modulation at $P_B = 10^{-5}$. For such MFSK, the IF minimum bandwidth, assuming minimum tone spacing, is given by

$$B = \frac{M}{T_s} = MR_s \tag{7.17}$$

where T_s is the symbol duration, and R_s is the symbol rate. With M-ary FSK, the required transmission bandwidth is expanded M-fold over binary FSK since there are M different orthogonal waveforms, each requiring a bandwidth of $\frac{1}{T_s}$. Thus, from Equations 7.14 and 7.17, the bandwidth efficiency of noncoherent MFSK signals can be expressed as

$$\frac{R}{B} = \frac{\log_2 M}{M} \ [bit/s/Hz] \tag{7.18}$$

Notice the important difference between the bandwidth efficiency$\left(\frac{R}{B}\right)$ of MPSK expressed in Equation (9.19) and that of MFSK expressed in Equation 7.16. With MPSK, $\frac{R}{B}$ increases as the signal dimensionality M increases. With MFSK there are two mechanisms at work. The numerator shows the same increase in$\frac{R}{B}$ with larger M, as in the case of MPSK. But the denominator indicates a decrease in $\frac{R}{B}$ with larger M. As M grows larger, the denominator grows faster than the numerator, and thus $\frac{R}{B}$ decreases. The MFSK points plotted in Figure 7.5 confirm the relationship shown in Equation 7.18, that orthogonal signaling such as MFSK is a bandwidth-expansive scheme. From Figure 7.5, it can be seen that MFSK modulation can be used for realizing a reduction in required $\frac{E_b}{N_0}$, at the cost of increased bandwidth.

It is important to emphasize that in Equations 7.15 and 7.16 for MPSK, and for all the MPSK points plotted in Figure 7.5, Nyquist (ideal rectangular) filtering has been assumed. Such filters are not realizable. For realistic channels and waveforms, the required transmission bandwidth must be increased in order to account for realizable filters. In each of the examples that follow, we consider radio channels, disturbed only by additive white Gaussian noise (AWGN) and having no other impairments. For simplicity, the modulation choice is limited to constant-envelope types-either MPSK or noncoherent orthogonal MFSK. Thus, for an uncoded system, if the channel is bandwidth limited, MPSK is selected, and if the channel is power limited, MFSK is selected. Note that, when error-correction coding is considered, modulation selection is not so simple, because there exist coding techniques that can provide power-bandwidth trade-offs more effectively than would be possible through the use of any M-ary modulation scheme.

Note that in the most general sense, M-ary signaling can be regarded as a waveform-coding procedure. That is, whenever we select an M-ary modulation technique instead of a binary one, we in effect have replaced the binary waveforms with better waveforms-either better for bandwidth performance (MPSK), or better for power performance (MFSK). Even though orthogonal MFSK signaling can be thought of as being a coded system (it can be described as a first-order Reed-Muller code), we shall here restrict our use of the term coded system to refer only to those traditional error-correction codes using redundancies, such as block codes or convolutional codes.

7.7.4 Requirements for MPSK and MFSK Signaling

The basic relationship between the symbol (or waveform) transmission rate R_s, and the data rate R was shown in Equation 7.13 to be

$$R_s = \frac{R}{\log_2 M}$$

Using this relationship together with Equations 7.14 through 7.17, and a given data rate of $R = 9600\ bit/s$, Table 7.1 has been compiled. The table is a summary of symbol rate, minimum bandwidth, and bandwidth efficiency for MPSK and noncoherent orthogonal MFSK, for the values of M = 2, 4, 8. 16, and 32. Also included in Table 9.1 are the required values of $\frac{E_b}{N_0}$ to achieve a bit-error probability of 10^{-5} for MPSK and MFSK for each value of M shown. These $\frac{E_b}{N_0}$ entries were computed using relationships that are presented later. The $\frac{E_b}{N_0}$ entries corroborate the trade-offs shown in Figure 7.5. As M increases, MPSK signaling provides more bandwidth efficiency at the cost of increased $\frac{E_b}{N_0}$, while MFSK signaling allows for a reduction in $\frac{E_b}{N_0}$ at the cost of increased bandwidth. The next three sections are presented in the context of examples taken from Table 7.1.

Table 7.1 Symbol Rate, Minimum Bandwidth, Bandwidth Efficiency, and Required $\frac{E_b}{N_0}$ for MPSK and Noncoherent Orthogonal MFSK Signaling at 9600 bit/s

M	**k**	**R** [bit/s]	$\boldsymbol{R_s}$ [symbol/s]	**MPSK Minimum Bandwidth** [Hz]	**MPSK** $\frac{R}{B}$	**MPSK** $\frac{E_b}{N_0}$ $[dB]$ $P_B = 10^{-5}$	**Noncoherent Orthog MFSK** Min Bandwidth [Hz]	**MFSK** $\frac{R}{B}$	**MFSK** $\frac{E_b}{N_0}$ $[dB]$ $P_B = 10^{-5}$
2	1	9600	9600	9600	1	9,6	19,200	1/2	13,4
4	2	9600	4800	4800	2	9,6	19,200	1/2	10,6
8	3	9600	3200	3200	3	13,0	25,600	1/3	9,1
16	4	9600	2400	2400	4	17,5	38,400	1/4	8,1
32	5	9600	1920	1920	5	22,4	61,440	5/32	7,4

7.7.5 Bandwidth-Limited Uncoded System Example

Suppose we are given a bandwidth-limited AWGN radio channel with an available bandwidth of $B = 4000\ Hz$. Also, consider that the link constraints (transmitter power, antenna gains, path loss, etc.) result in the ratio of received signal power to noise-power spectral density $\left(\frac{P_r}{N_0}\right)$ being equal to 53 dB/Hz. Let the required data rate R be equal to $9600\ bits/s$, and let the required bit-error performance P_B be at most 10^{-5}. The goal is to choose a modulation scheme that meets the required performance. In general, an error-correction coding scheme may be needed if none of the allowable modulation schemes can meet the requirements. However, in this example, we will see that the use of error-correction coding is not necessary.

For any digital communication system, the relationship between received power to noise-power spectral density $\left(\frac{P_r}{N_0}\right)$ and received bit-energy to noise power spectral density $\left(\frac{E_b}{N_0}\right)$ to be

$$\frac{P_r}{N_0} = \frac{E_b}{N_0} R \tag{7.19}$$

Solving for $\frac{E_b}{N_0}$ in decibels, we obtain

$$\begin{aligned} \frac{E_b}{N_0}[dB] &= \frac{P_r}{N_0} - R \\ &= 53 - (10\log_{10} 9600) = 13{,}2\ dB\ (or\ 20{,}89) \end{aligned} \tag{7.20}$$

Since the required data rate of 9600 bits/s is much larger than the available bandwidth of 4000 Hz, the channel can be described as bandwidth limited. We therefore select MPSK as our modulation scheme. Recall that we have confined the possible modulation choices to be constant-envelope types: without such a restriction, it would be possible to select a modulation type with greater bandwidth efficiency. In an effort to conserve power, we next compute the smallest possible value of M, such that the symbol rate is at most equal to the available bandwidth of 4000 Hz. From Table 7.1, it is clear that the smallest value of M meeting this requirement is M = 8. Our next task is to determine whether the required bit-error performance of $P_B \leq 10^{-5}$ can be met by using 8-PSK modulation alone, or whether it is necessary to additionally use an error-correction coding scheme. It can be seen from Table 7.1. that 8-PSK alone will meet the requirements, since the required $\frac{E_b}{N_0}$ listed for 8-PSK is less then the received $\frac{E_b}{N_0}$ that was derived in Equation 7.20. However, imagine that we do not have Table 7.1. Let us demonstrate how to evaluate whether or not error-correction coding is necessary. Figure 7.7 shows the basic modulator/demodulator (MODEM) block diagram summarizing the functional details of this design. At the modulator, the transformation from data bits to symbols yields an output symbol rate R_s that is a factor $(\log_2 M)$ smaller than the input data-bit rate R, as can be seen in Equation 7.13. Similarly, at the input to the demodulator, the symbol-energy to noise-power spectral density $\frac{E_b}{N_0}$ is a factor $(\log_2 M)$ larger than $\frac{E_b}{N_0}$, since each symbol is made up of $(\log_2 M)$ bits. Because $\frac{E_s}{N_0}$ is larger than $\frac{E_b}{N_0}$ by the same factor that R_s, is smaller than R, we can expand Equation 7.19, as follows:

$$\frac{P_r}{N_0} = \frac{E_b}{N_0} R = \frac{E_s}{N_0} R_s \tag{7.21}$$

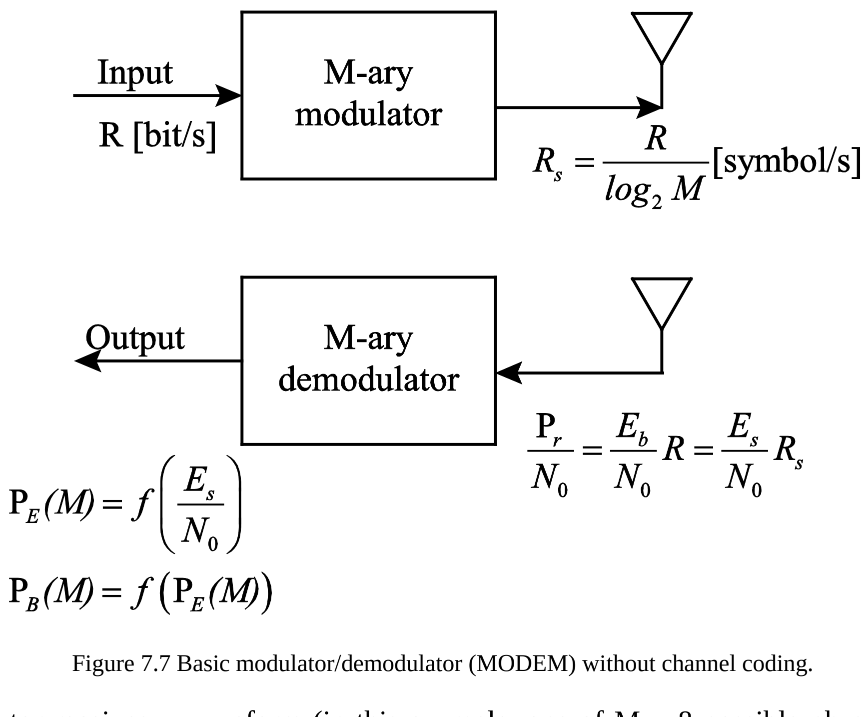


Figure 7.7 Basic modulator/demodulator (MODEM) without channel coding.

The demodulator receives a waveform (in this example, one of M = 8 possible phase shifts) during each time interval T_s. The probability that the demodulator makes a symbol error $P_E(M)$ is well approximated and we write

$$P_E(M) \approx 2Q\left[\sqrt{\frac{2E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right] for\ M > 2 \tag{7.22}$$

where $Q(x)$, the complementary error function, is

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty exp\left(-\frac{u^2}{2}\right)du \tag{7.23}$$

In Figure 7.6 and all the figures that follow, rather than show explicit probability relationships, the generalized notation $f(x)$ has been used to indicate some functional dependence on x.

A traditional way of characterizing communication (power) efficiency or error performance in digital systems is in terms of the received $\frac{E_b}{N_0}$ in decibels. This $\frac{E_b}{N_0}$ description has become standard practice. However, recall that at the input to the demodulator/detector, there are no bits; there are only waveforms that have been assigned bit meanings. Thus, the received $\frac{E_b}{N_0}$ value represents a bitapportionment of the arriving waveform energy. A more precise (but unwieldy) name would be the energy per effective bit versus N_0. To solve for $P_E(M)$ in Equation 7.22, we first need to compute the ratio of received symbol-energy to noise power spectral density, $\frac{E_s}{N_0}$. Since, from Equation 7.20, $\frac{E_b}{N_0} = 13{,}2\ dB\ (or\ 20{,}89)$, and because each symbol is made up of $(\log_2 M)$ bits, we compute, with M=8,

$$\frac{E_s}{N_0} = (\log_2 M)\frac{E_b}{N_0} = 3 * 20{,}89 = 62{,}67 \tag{7.24}$$

Using the results of Equation 7.24 in Equation 7.23, yields the symbol-error probability, $P_E = 2{,}2.10^{-5}$. To transform this to bit-error probability. we need to use the relationship between bit-error probability P_B and symbol-error probability P_E for multiple-phase signaling. We write

$$P_B \approx \frac{P_E}{\log_2 M}\ (for\ P_E \ll 1) \tag{7.25}$$

which is a good approximation, when Gray coding is used for the bit-to-symbol assignment. This last computation yields $P_B = 7{,}3.10^{-6}$, which meets the required bit-error performance. Thus, in this example, no error-correction coding is necessary and 8-PSK modulation represents the design choice to meet the requirements of the bandwidth-limited channel (which we had predicted by examining the required $\frac{E_b}{N_0}$ values in Table 7.1).

7.7.6 Power-Limited Uncoded System Example

Now, suppose that we have exactly the same data rate and bit-error probability requirements as in the example of Chapter 7.7.5. However, in this example, let the available bandwidth W be equal to 45 kHz, and let the available $\frac{P_r}{N_0}$ be equal to 48 dB/Hz. As before, the goal is to choose a modulation or modulation/coding scheme that yields the required performance. In this example, we shall again find that error-correction coding is not required.

The channel in this example is clearly not bandwidth limited since the available bandwidth of 45 kHz is more than adequate for supporting the required data rate of 9600 bits/s. The received $\frac{E_b}{N_0}$ is found from Equation 7.20, as follows:

$$\frac{E_b}{N_0}[dB] = 48 - (10\log_{10} 9600) = 8{,}2\ dB\ (or\ 6{,}61) \tag{7.26}$$

Since there is abundant bandwidth but a relatively small amount of $\frac{E_b}{N_0}$ for the required bit-error probability, this channel may be referred to as power limited. We therefore choose MFSK as the modulation scheme. In an effort to conserve power, we next search for the largest possible M such that the MFSK minimum bandwidth is not expanded beyond our available bandwidth of 45 kHz. From Table 7.1, we see that such a search results in the choice of M = 16, our next task is to determine whether the required error performance of $P_B \leq 10^{-5}$ can be met by using 16-FSK alone, without the use of any error-correction coding. Similar to the previous example, it can be seen from Table 7.1, that 16-FSK alone will meet the requirements, since the required $\frac{E_b}{N_0}$ listed for 16-FSK is less than the received $\frac{E_b}{N_0}$ that was derived in Equation 7.26. However, imagine again that we do not have Table 7.1. Let us demonstrate how to evaluate whether or not error-correction coding is necessary.

As before, the block diagram in Figure 7.7 summarizes the relationship between symbol rate R_s and bit rate R, and between $\frac{E_s}{N_0}$ and $\frac{E_b}{N_0}$, which is identical to each of the respective relationships in the previous bandwidth-limited example. In this example, the 16-FSK demodulator receives a waveform (one of 16 possible frequencies) during each symbol time interval T_s. For noncoherent MFSK, the probability that the demodulator makes a symbol error is approximated by

$$P_E(M) \leq \frac{M-1}{2} exp\left(-\frac{E_s}{2N_0}\right) \tag{7.27}$$

To solve for $P_E(M)$ in Equation 9.27, we need to compute $\frac{E_s}{N_0}$, as we did in Example 1. Using the results of Equation 7.26 in Equation 7.24, with M = 16, we get

$$\frac{E_s}{N_0} = (\log_2 M)\frac{E_b}{N_0} = 4 * 6{,}61 = 26{,}44 \tag{7.28}$$

Next, we combine the results of Equation 7.28 in Equation 7.27 to yield the symbol-error probability $P_E = 1{,}4.10^{-5}$. To transform this to bit-error probability P_B, we need to use the relationship between P_B and P_E for orthogonal signaling, given by

$$P_B = \frac{2^{k-1}}{2^k - 1} P_E \tag{7.29}$$

This last computation yields $P_B = 7{,}3.10^{-6}$, which meets the required bit-error performance. Thus, we can meet the given specifications for this power-limited channel by using 16-FSK modulation, without any need for error-correction coding (which we had predicted by examining the required $\frac{E_b}{N_0}$ values in Table 7.1).

7.8 SOLVED PROBLEMS

Problem 1

In some sense, all digital modulation schemes fall into one of two classes with opposite behavior characteristics. The first class constitutes orthogonal signaling, and its error performance follows the curves shown in Figure 7.1 a). The second class constitutes nonorthogonal signaling (the constellation of signal phasors can be depicted on a plane). Figure 7.1 b) illustrates an MPSK example of such nonorthogonal signaling. However, any phase/amplitude modulation (e.g., QAM) falls into this second class. In the context of Figure 7.1, answer the following questions:

a) Does error-performance improve or degrade with increasing M, for M-ary signaling?
b) The choices available in digital communications almost always involves a tradeoff. If error-performance improves, what price must we pay?
c) If error-performance degrades, what benefit is exhibited?

Solution

a) When examining Figure 7.1, we see that error-performance improvement or degradation depends upon the class of signaling in question. Consider the orthogonal signaling in Figure 7.1 a), where error-performance improves with increased k or M. Recall that there are only two fair ways to compare error-performance with such curves. A vertical line can be drawn through some fixed value of $\frac{E_b}{N_0}$ and ask or M is increased, it is seen that P_B is reduced, or, a horizontal line can be drawn through some fixed P_B requirement, and ask or M is increased, it is seen that the $\frac{E_b}{N_0}$ requirement is reduced. Similarly, it can be seen that the curves in Figure 7.1 b) for nonorthogonal signaling such as MPSK, behave in the opposite fashion. Error-performance degrades ask or M is increased.

b) In the case of orthogonal signaling, where error performance improves with increasing k of M, what is the cost? In terms of the orthogonal signaling we are most familiar with, MFSK, when $k = 1$ and $M = 2$ there are two tones in the signaling set. When $k = 2$ and $M = 4$, there are four tones in the set. When $k = 3$ and $M = 8$. There are eight tones, and so forth. With MFSK, only one tone is sent during each symbol time, but the available transmission bandwidth consists of the entire set of tones. Hence, as k or M is increased, it should be clear that the cost of improved error-performance is an expansion of required bandwidth.

c) In the case of nonorthogonal signaling, such as MPSK or QAM, where error performance degrades as k or M is increased, one might rightfully guess that the tradeoff will entail a reduction in the required bandwidth. Consider the following example. Suppose we require a data rate of $R = 9600\ bit/s$. And, suppose that the modulation chosen is 8-ary PSK. Then, using Equation 7.1, we find that the symbol rate is

$$R_s = \frac{R}{\log_2 M} = \frac{9600\ {}^{bit}/_{s}}{3\ {}^{bit}/_{symbol}} = 3200\ {}^{symbol}/_{bit}$$

If we decide to use 16-ary PSK for this example, the symbol rate would then be

$$R_s = \frac{R}{\log_2 M} = \frac{9600\, {bit}/{s}}{4\, {bit}/{symbol}} = 2400\, {symbol}/{bit}$$

If we continue in this direction and use 32-ary PSK, the symbol rate becomes

$$R_s = \frac{R}{\log_2 M} = \frac{9600\, {bit}/{s}}{59\, {bit}/{symbol}} = 1920\, {symbol}/{bit}$$

Do you see what happens as the operating point in Figure 7.1 b) is moved along a horizontal line from the $k = 3$ curve to the $k = 4$ curve. and finally to the $k = 5$ curve? For a given data rate and bit-error probability, each such movement allows us to signal at a slower rate. Whenever you hear the words. "*slower signaling rate*," that is tantamount to saying that the transmission bandwidth can be reduced. Similarly, any case of increasing the signaling rate, corresponds to a need for increasing the transmission bandwidth.

Problem 2

Suppose $P_B = 10^{-6}$ is desired for a certain digital data transmission system.

a) Compare the necessary SNRs for BPSK, DPSK, antipodal PAM for $M = 2, 4, 8;$ and noncoherent FSK.
b) Compare maximum bit rates for an RF bandwidth of 20 kHz.

Solution

For part a), we find by trial and error that $Q(4{,}753) \approx 10^{-6}$. Biphase-shift keying and antipodal PAM for $M = 2$have the same bit error probability, given by

$$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = 10^{-6}$$

So that $\sqrt{\frac{2E_b}{N_0}} = 4{,}753$ or $\frac{E_b}{N_0} = \frac{(4{,}753)^2}{2} = 11{,}3 = 10{,}53dB$. Then for M=4:

$$\frac{2(4-1)}{4\log_2 4} Q\left(\sqrt{\frac{6\log_2 4}{4^2 - 1}\frac{E_b}{N_0}}\right) = 10^{-6}$$

$$Q\left(\sqrt{0{,}8\frac{E_b}{N_0}}\right) = 1{,}333 * 10^{-6}$$

Another trial-and-error search gives $Q(4{,}753) \approx 1{,}333 * 10^{-6}$ so that $\sqrt{0{,}8\frac{E_b}{N_0}} = 4{,}695$ or $\frac{E_b}{N_0} = \frac{(4{,}495)^2}{0{,}8} = 27.55 = 14{,}4\ dB$. For M=8

$$\frac{2(4-1)}{8\log_2 8} Q\left(\sqrt{\frac{6\log_2 8}{8^2 - 1}\frac{E_b}{N_0}}\right) = 10^{-6}$$

$$\frac{7}{12}Q\left(\sqrt{\frac{2}{7}\frac{E_b}{N_0}}\right) = 10^{-6}$$

$$Q\left(\sqrt{0{,}268\frac{E_b}{N_0}}\right) = 1{,}714 * 10^{-6}$$

Another trial-and-error search gives $Q(4{,}642) \approx 1{,}714 * 10^{-6}$ so that $\sqrt{0{,}286\frac{E_b}{N_0}} = 4{,}643$ or $\frac{E_b}{N_0} = \frac{(4{,}643)^2}{0{,}286} = 75{,}38 = 18{,}77\ dB$.

For DPSK, we have>

$$\frac{1}{2}e^{-\frac{E_b}{N_0}} = 10^{-6}$$

$$e^{-\frac{E_b}{N_0}} = 2 * 10^{-6}$$

$$\frac{E_b}{N_0} = -\ln(2 * 10^{-6}) = 13{,}12 = 11{,}18\ dB$$

For coherent FSK, we have

$$P_B = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = 10^{-6}$$

So that

$$\sqrt{\frac{E_b}{N_0}} = 4{,}753\ or\ \frac{E_b}{N_0} = (4{,}753)^2 = 22{,}59 = 13{,}54dB$$

For noncoherent FSK, we have

$$\frac{1}{2}e^{-\frac{1}{2}\frac{E_b}{N_0}} = 10^{-6}$$

$$e^{-\frac{1}{2}\frac{E_b}{N_0}} = 2 * 10^{-6}$$

$$\frac{E_b}{N_0} = -2\ln(2 * 10^{-6}) = 26{,}24 = 14{,}18\ ddB$$

For b), we use the previously developed bandwidth expressions and results are given in Table. The results of Table demonstrate that PAM is a modulation scheme that allows a trade-off between power efficiency (in terms of the $\frac{E_b}{N_0}$ required for a desired bit-error probability) and bandwidth efficiency (in terms of maximum data rate for a fixed bandwidth channel).

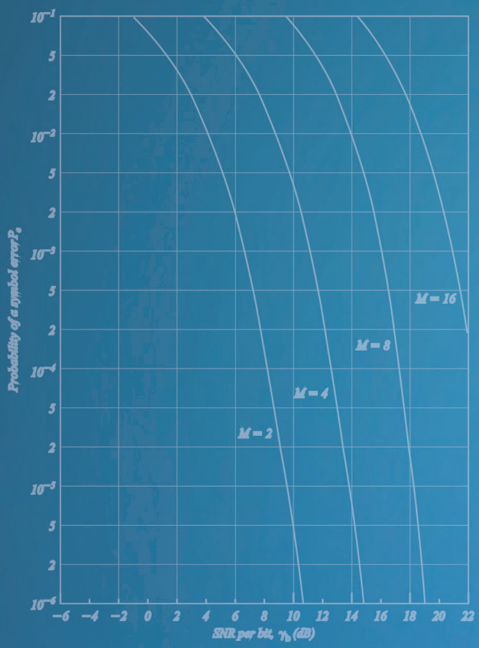
Modulation method	**Required SNR for $P_B = 10^{-6}$ $[dB]$**	**R for $B_{RF} = 20kHz$ $[kbps]$**
BPSK	10,5	10
DPSK	11,2	10
Antipodal 4-PAM	14,4	20
Antipodal 8-PAM	18,8	30
Coherent FSK, ASK	13,5	8
Noncoherent FSK	14,2	5

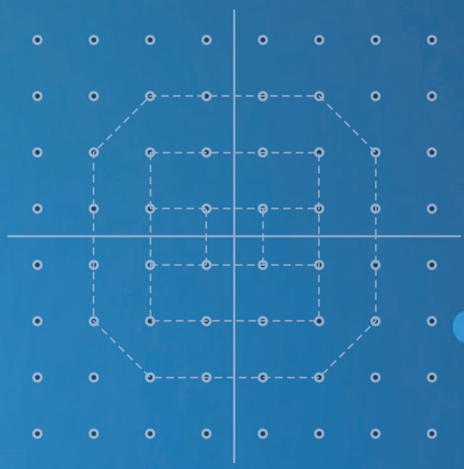
7.9 SUMMARY

- When dealing with M-ary digital communications systems, with $\mathrm{M} \geq 2$ it is important to distinguish between a bit and a symbol or character. A symbol conveys $\log_2 \mathrm{M}$ bits.We must also distinguish between bit-error probability and symbol-error probability.
- M-ary schemes based on quadrature multiplexing include QPSK, OQPSK, andMSK. All have a bit-error rate performance that is essentially the same as binaryBPSK if precoding is used to ensure that only one bit error results from mistaking a given phase for an adjacent phase.
- Minimum-shift keying can be produced by quadrature modulation or by serial modulation. In the latter case,MSKis produced by filtering BPSK with a properly designed conversion filter. At the receiver, serial MSK can be recovered by first filtering it with a bandpass matched filter and performing coherent demodulation with a carrier at $f_c + \frac{1}{4}T_b$ (i.e., at the carrier plus a quarter data rate). Serial MSK performs identically to quadrature-modulated MSK and has advantageous implementation features at high data rates
- It is convenient to view M-ary data modulation in terms of signal space. Examples of data formats that can be considered in this way are M-ary PSK, QAM, and M-ary FSK. For the former two modulation schemes, the dimensionality of the signal space stays constant as more signals are added; for the latter, it increases directly as the number of signals added. A constant-dimensional signal space means signal points are packed closer as the number of signal points is increased, thus degrading the error probability; the bandwidth remains essentially constant. In the case of FSK, with increasing dimensionality as more signals are added, the signal points are not compacted, and the error probability decreases for a constant SNR; the bandwidth increases with an increasing number of signals, however.
- Communication systems may be compared on the basis of power and bandwidth efficiencies. A rough measure of bandwidth is null-to-null of the main lobe of the transmitted signal spectrum. For M-ary PSK, QAM, and DPSK power efficiency decreases with increasing M (i.e., as M increases a larger value of $\frac{E_b}{N_0}$is required to provide a given value of bit-error probability) and bandwidth efficiency increases (i.e, the larger M, the smaller the required bandwidth for a given bit rate). For M-ary FSK (both coherent and noncoherent) the opposite is true. This behavior may be explained with the aid of signal space concepts—the signal space for M-ary PSK, QAM, and DPSK remains constant at two dimensions versus M (one-dimensional for $M = 2$), whereas for M-ary FSK it increases linearly with M. Thus, from a power efficiency standpoint the signal points are crowded together more as M increases in the former cases, whereas they are not in the latter case.

7.10 EXERCISE

1. An M-ary communication system transmits at a rate of 4000 symbols per second. What is the equivalent bit rate in bits per second for $M = 4, M = 8, M = 16, M = 32\ and\ M = 64$ Generate a plot of bit rate versus $\log_2 M$.
2. A speech signal is sampled at a rate of 8 KHz, logarithmically compressed and encoded into a PCM format using 8 bits per sample. The PCM data is transmitted through an AWGN baseband channelvia M-level PAM signaling. Determine the required transmission bandwidth when (a) M = 4, (b) M = 8 and (c) M = 16. (Assume rectangular pulses and the zero-to-null definition of bandwidth.)
3. Binary PSK (BPSK) is used for data transmission over an AWGN channel with power spectral density $\frac{N_0}{2} = 10^{-10}$ W/Hz. The transmitted signal energy is $E_b = \frac{A^2T}{2}$, where T is the bit duration and A is the signal amplitude. Determine the value of A needed to achieve an error probability of 10−6, if the data rate is:
   a. 10 Kbit/s
   b. 100 Kbit/s
   c. 1 Mbit/s
4. You are required to provide a real-time communication system to support 9600 bits/s with a required bit-error probability of at most 10^{-5} within an available bandwidth of 2700 Hz. The predetection $\frac{P_r}{N_0}$ is 54.8 dB-Hz. Choose one of two modulation schemes-either MPSK with Gray coding or noncoherent orthogonal MFSK. Such that the available bandwidth is not exceeded and power is conserved.
5. A digital communication system transmits data using QAM signaling over a voice-band telephone channel at a rate 2400 symbols/s (baud). The additive noise is assumed to be white and Gaussian. You are asked to determine the energy-per-bit-to-noise ratio $\frac{E_b}{N_0}$ required to achieve an error probability of 110^{-5} for a bit rate equal to:
   a. 4800 bits/s
   b. 9600 bits/s
   c. 19200 bits/s
   d. 31200 bits/s
6. Assuming that it is desired to transmit information at the rate of R [bits/s], determine the required transmission bandwidth of each of the following six communication systems, and arrange them in order of bandwidth efficiency, starting from the most bandwidth-efficient and ending at the least bandwidth-efficient.





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