Why $N = \lfloor 2WT + 1 \rfloor$?

- For signals with bandwidth W, the Nyquist rate is $2W$ for perfect reconstruction.
- You then get $2W$ samples/second.
  $\Longrightarrow$ $2W$ degrees of freedom (per second)
- For time duration T, you get overall $2WT$ samples.
  $\Longrightarrow$ $2WT$ degrees of freedom (per T seconds)

Bandwidth efficiency (Simplified view)

$$N = 2WT \Rightarrow \frac{1}{W} = \frac{2T}{N}$$

Since rate $R = \frac{1}{T} \times \log_2 M$, we have for M-ary signaling

$$\frac{R}{W} = \frac{\log_2(M)}{T}\frac{2T}{N} = 2\frac{\log_2(M)}{N}$$

where

- $\log_2(M)$ is the number of bits transmitted at a time
- N is **usually** (see SSB PAM and DSB PAM as counterexamples) the dimensionality of the constellation

Thus $\frac{R}{W}$ can be regarded as bit/dimension (it is actually measured as bit per second per Hz).

R/W is called bandwidth efficiency.

The dimensionality theorem helps us to derive a relation between bandwidth and dimensionality of a signaling scheme. If the set of signals in a signaling scheme consists of M signals each with duration T_s, the signaling interval, and the approximate bandwidth of the set of signals is W, the dimensionality of the signal space is $N = 2WT_s$.

Using the relation $R_s = 1/T_s$, we have

$$W = \frac{R_s N}{2} \tag{4.6–6}$$

Since $R = R_s \log_2 M$, we conclude that

$$W = \frac{RN}{2 \log_2 M} \tag{4.6–7}$$

and

$$r = \frac{R}{W} = \frac{2 \log_2 M}{N} \tag{4.6–8}$$

This relation gives the bandwidth efficiency of a signaling scheme in terms of the constellation size and the dimensionality of the constellation.

In one-dimensional modulation schemes (ASK and PAM), $N = 1$ and $r = 2\log_2 M$. PAM and ASK can be transmitted as single-sideband (SSB) signals.

For two-dimensional signaling schemes such as QAM and MPSK, we have $N = 2$ and $r = \log_2 M$. It is clear from the above discussion that in MASK, MPSK, and MQAM signaling schemes the bandwidth efficiency increases as M increases. As we have seen before in all these systems, the power efficiency decreases as M is increased. Therefore, the size of constellation in these systems determines the tradeoff between power and bandwidth efficiency. These systems are appropriate where we have limited bandwidth and desire a bit rate–to–bandwidth ratio $r > 1$ and where there is sufficiently high SNR to support increases in M. Telephone channels and digital microwave radio channels are examples of such band-limited channels.

For M-ary orthogonal signaling, $N = M$ and hence Equation 4.6–8 results in

$$r = \frac{2\log_2 M}{M} \tag{4.6–9}$$

Obviously in this case as M increases, the bandwidth efficiency decreases, and for large M the system becomes very bandwidth-inefficient. Again as we had seen before in orthogonal signaling, increasing M improves the power efficiency of the system, and in fact this system is capable of achieving the Shannon limit as M increases. Here again the tradeoff between bandwidth and power efficiency is clear. Consequently, M-ary orthogonal signals are appropriate for power-limited channels that have sufficiently large bandwidth to accommodate a large number of signals. One example of such channels is the deep space communication channel.

N= dimension of modulation
(number of orthonormal function)

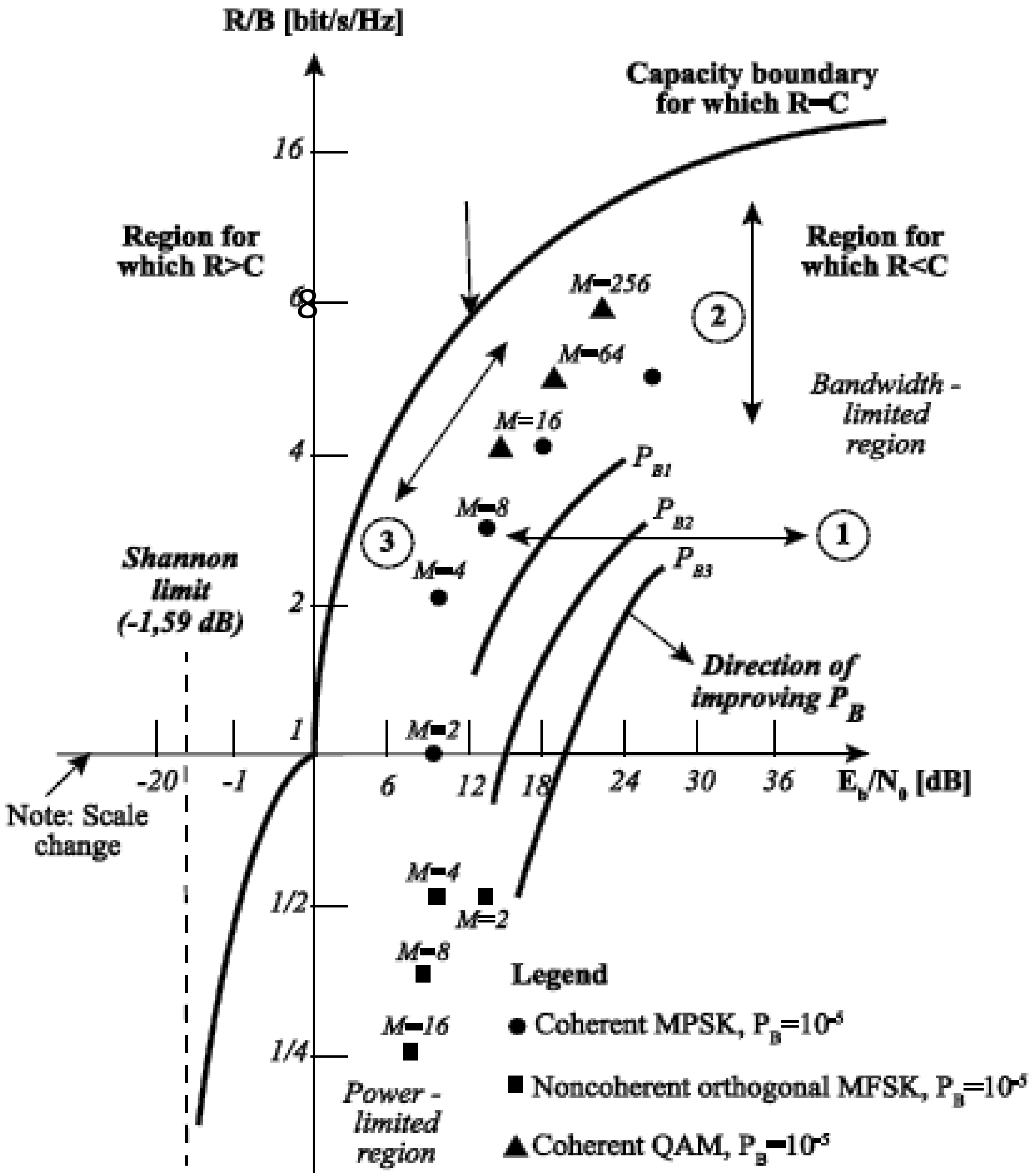
In general r=R/B = $2log_2(M)/N$

APSK, PAM --→ N=1; r=$2log_2(M)$

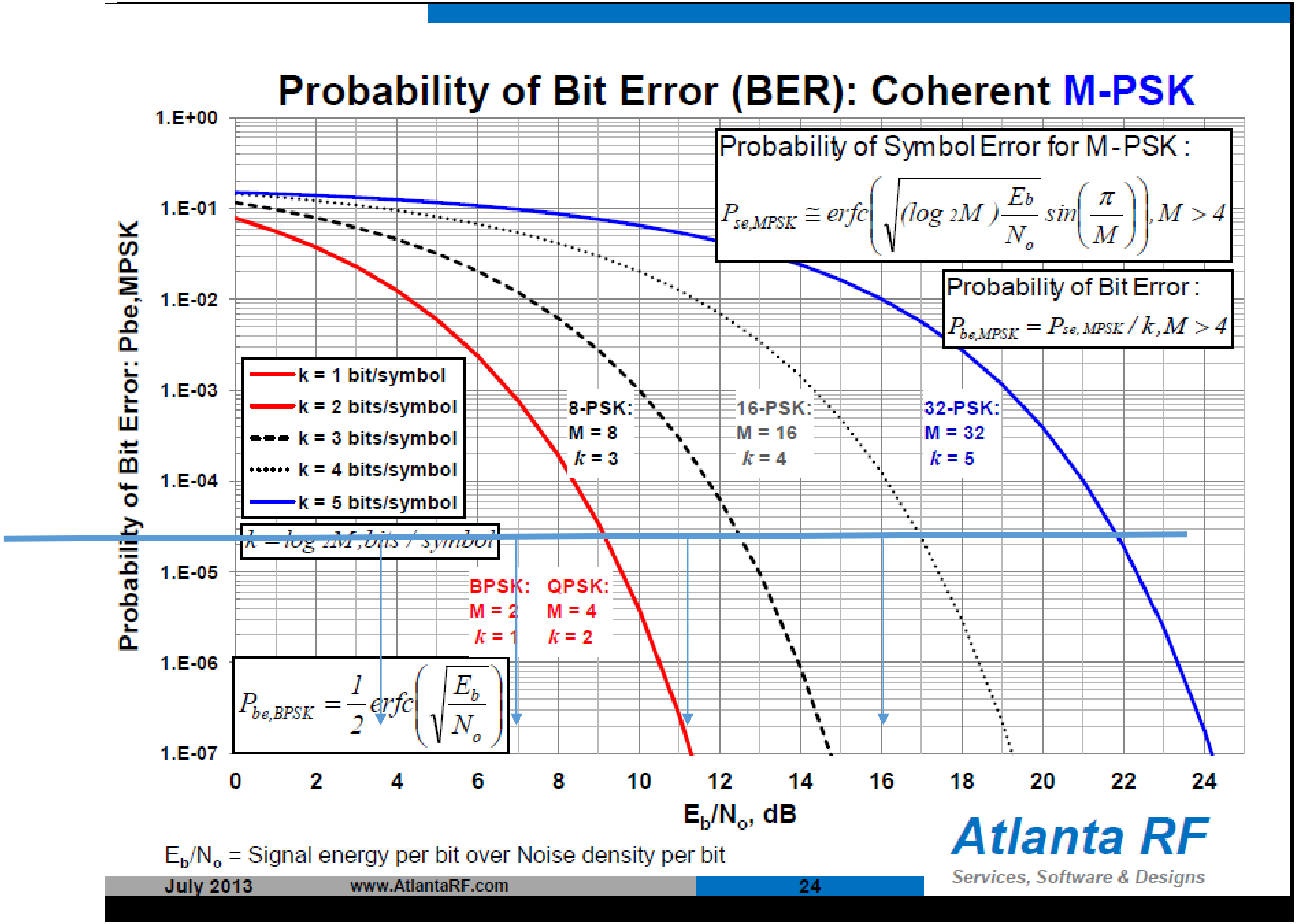
QAM, MPSK --→ N=2; r=$log_2(M)$

FSK (coherent) ; r=$2log_2(M)/M$

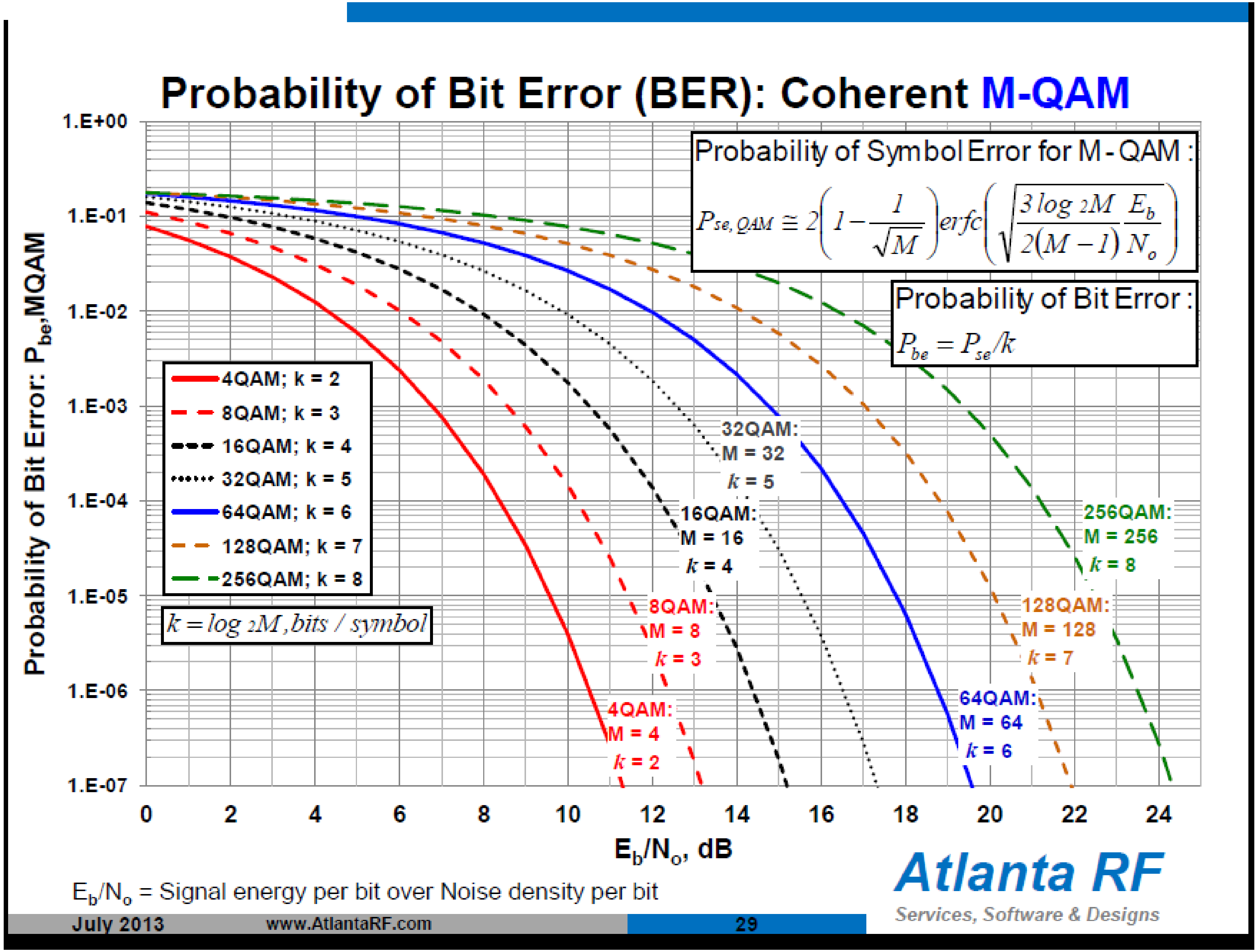
FSK (noncoherent) ; r=$log_2(M)/M$



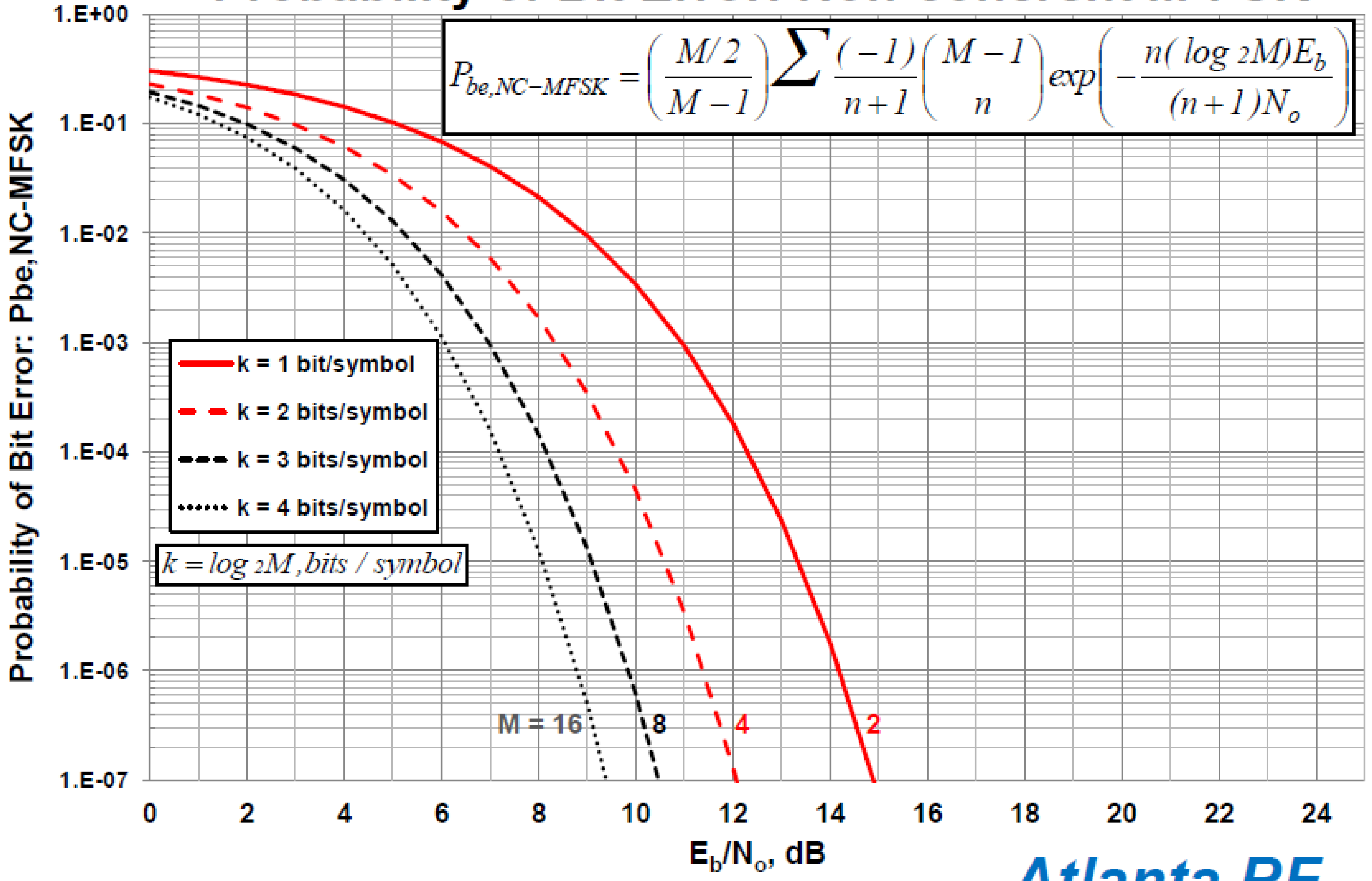
$$P_{\mathrm{BER-MPSK}} \approx \frac{2}{\log_2 M} Q\left(\sqrt{\frac{2E_b \log_2 M}{N_0}} \sin\left(\frac{\pi}{M}\right)\right)$$



$$P_{\mathrm{BER-QAM}} \approx \left(\frac{4\left(\sqrt{M}-1\right)}{\sqrt{M}\log_2 M}\right) Q\left(\sqrt{\frac{3\log_2 M}{M-1}\frac{E_b}{N_0}}\right)$$



Probability of Bit Error: Non-coherent M-FSK



E_b/N_o = Signal energy per bit over Noise density per bit

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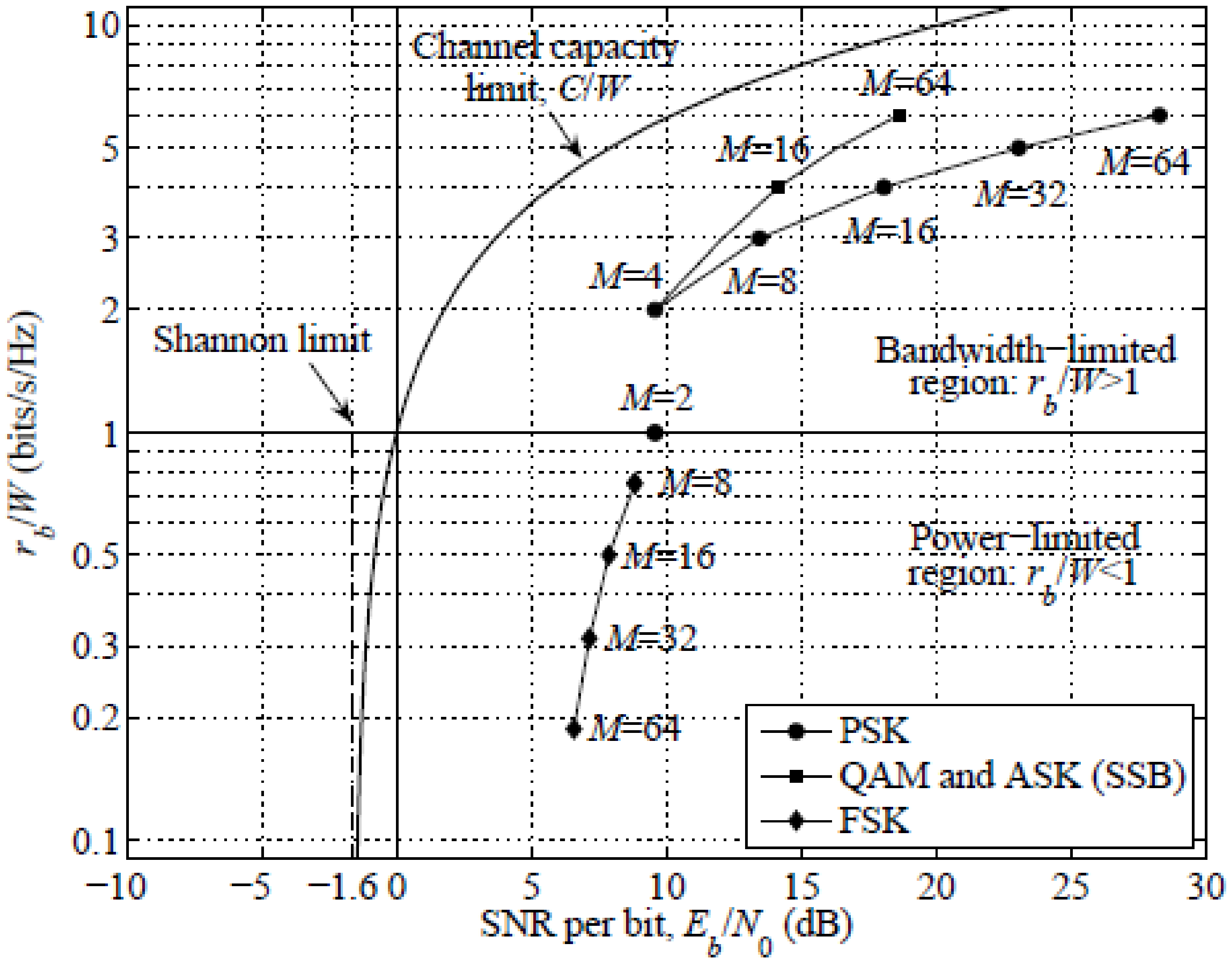
Shannon's Capacity Curve $\frac{E_b}{N_0} = \frac{2^{C/W}-1}{C/W}$

MPSK, QAM coherent

R/B = $log_2(M)$

MFSK noncoherent orthogonal

R/B = $log_2(M)/M$



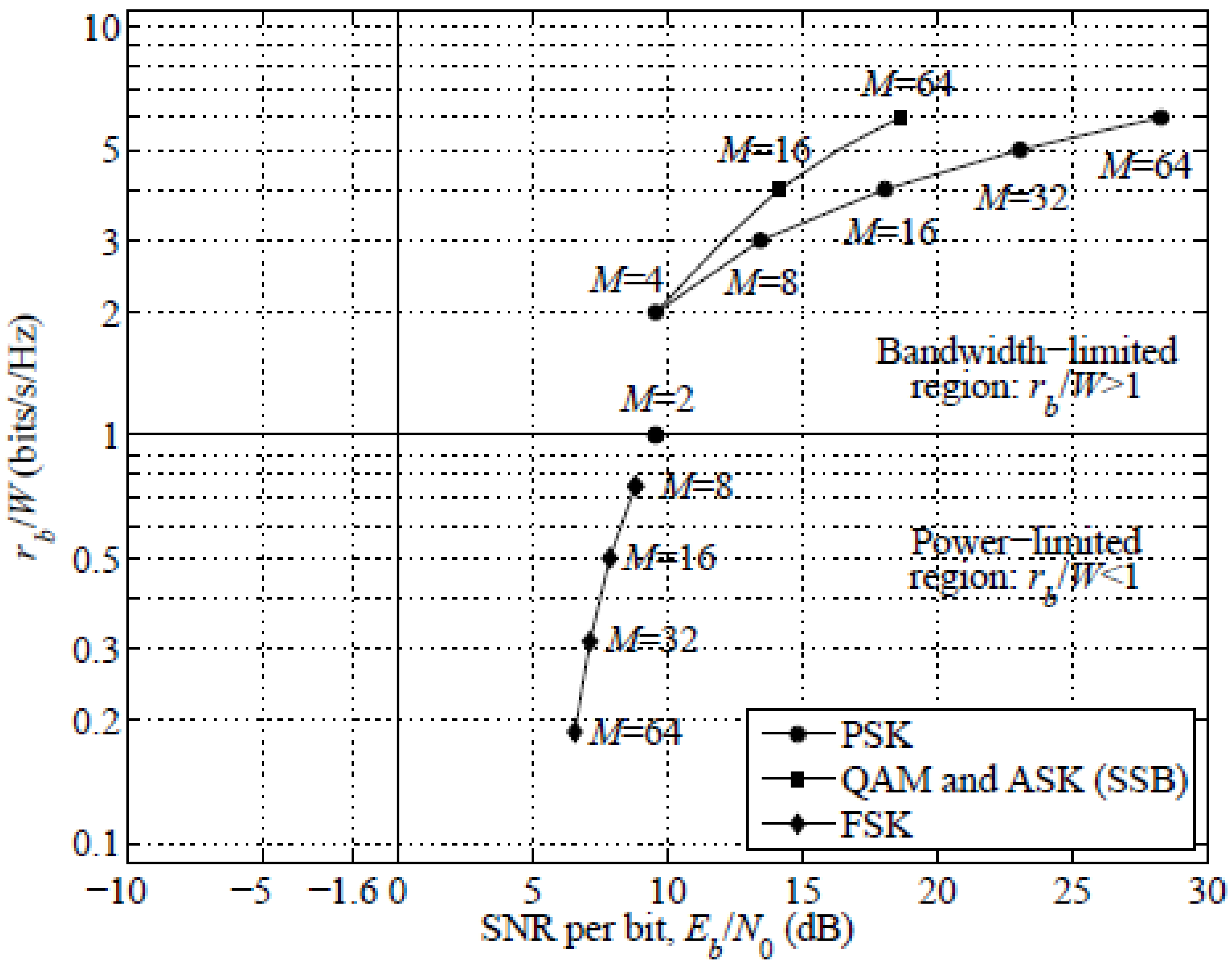
Power-Bandwidth Plane (At $P[\text{error}] = 10^{-5}$)

MPSK, QAM coherent

R/B = $log_2(M)$

MFSK noncoherent orthogonal

R/B = $log_2(M)/M$



MPSK, QAM coherent

R/B = $log_2(M)$

MFSK noncoherent orthogonal

R/B = $log_2(M)/M$

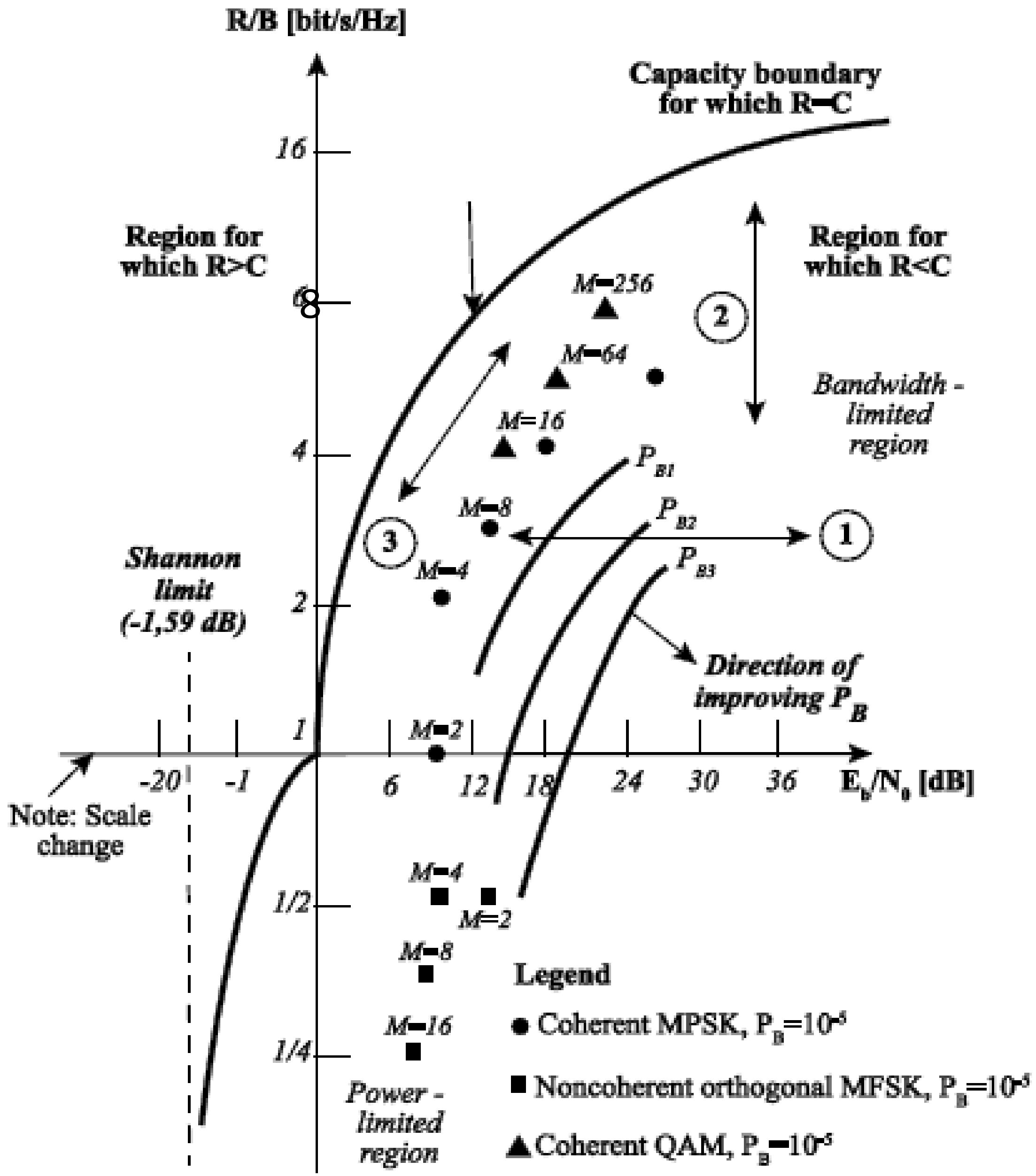


TABLE 2.3–2
Selected Q Function Values

$Q(x)$	x
10^{-1}	1.2816
10^{-2}	2.3263
10^{-3}	3.0902
10^{-4}	3.7190
10^{-5}	4.2649
10^{-6}	4.7534
10^{-7}	5.1993
0.5×10^{-5}	4.4172
0.25×10^{-5}	4.5648
0.667×10^{-5}	4.3545

TABLE 2.3–1
Table of Q Function Values

x	$Q(x)$	x	$Q(x)$	x	$Q(x)$	x	$Q(x)$
0	0.500000	1.8	0.035930	3.6	0.000159	5.4	3.3320×10^{-8}
0.1	0.460170	1.9	0.028717	3.7	0.000108	5.5	1.8990×10^{-8}
0.2	0.420740	2	0.022750	3.8	7.2348×10^{-5}	5.6	1.0718×10^{-8}
0.3	0.382090	2.1	0.017864	3.9	4.8096×10^{-5}	5.7	5.9904×10^{-9}
0.4	0.344580	2.2	0.013903	4	3.1671×10^{-5}	5.8	3.3157×10^{-9}
0.5	0.308540	2.3	0.010724	4.1	2.0658×10^{-5}	5.9	1.8175×10^{-9}
0.6	0.274250	2.4	0.008198	4.2	1.3346×10^{-5}	6	9.8659×10^{-10}
0.7	0.241960	2.5	0.006210	4.3	8.5399×10^{-6}	6.1	5.3034×10^{-10}
0.8	0.211860	2.6	0.004661	4.4	5.4125×10^{-6}	6.2	2.8232×10^{-10}
0.9	0.184060	2.7	0.003467	4.5	3.3977×10^{-6}	6.3	1.4882×10^{-10}
1	0.158660	2.8	0.002555	4.6	2.1125×10^{-6}	6.4	7.7689×10^{-11}
1.1	0.135670	2.9	0.001866	4.7	1.3008×10^{-6}	6.5	4.0160×10^{-11}
1.2	0.115070	3	0.001350	4.8	7.9333×10^{-7}	6.6	2.0558×10^{-11}
1.3	0.096800	3.1	0.000968	4.9	4.7918×10^{-7}	6.7	1.0421×10^{-11}
1.4	0.080757	3.2	0.000687	5	2.8665×10^{-7}	6.8	5.2309×10^{-12}
1.5	0.066807	3.3	0.000483	5.1	1.6983×10^{-7}	6.9	2.6001×10^{-12}
1.6	0.054799	3.4	0.000337	5.2	9.9644×10^{-8}	7	1.2799×10^{-12}
1.7	0.044565	3.5	0.000233	5.3	5.7901×10^{-8}	7.1	6.2378×10^{-13}

Example 1:
2 – PSK, PAM binary antipodal

$$P_{be} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$10^{-5} = Q\ (4{,}2649)$
Eb/N0 = (4,2649)^2/2
Eb/N0 = 10log(Eb/N0)=9,5 dB

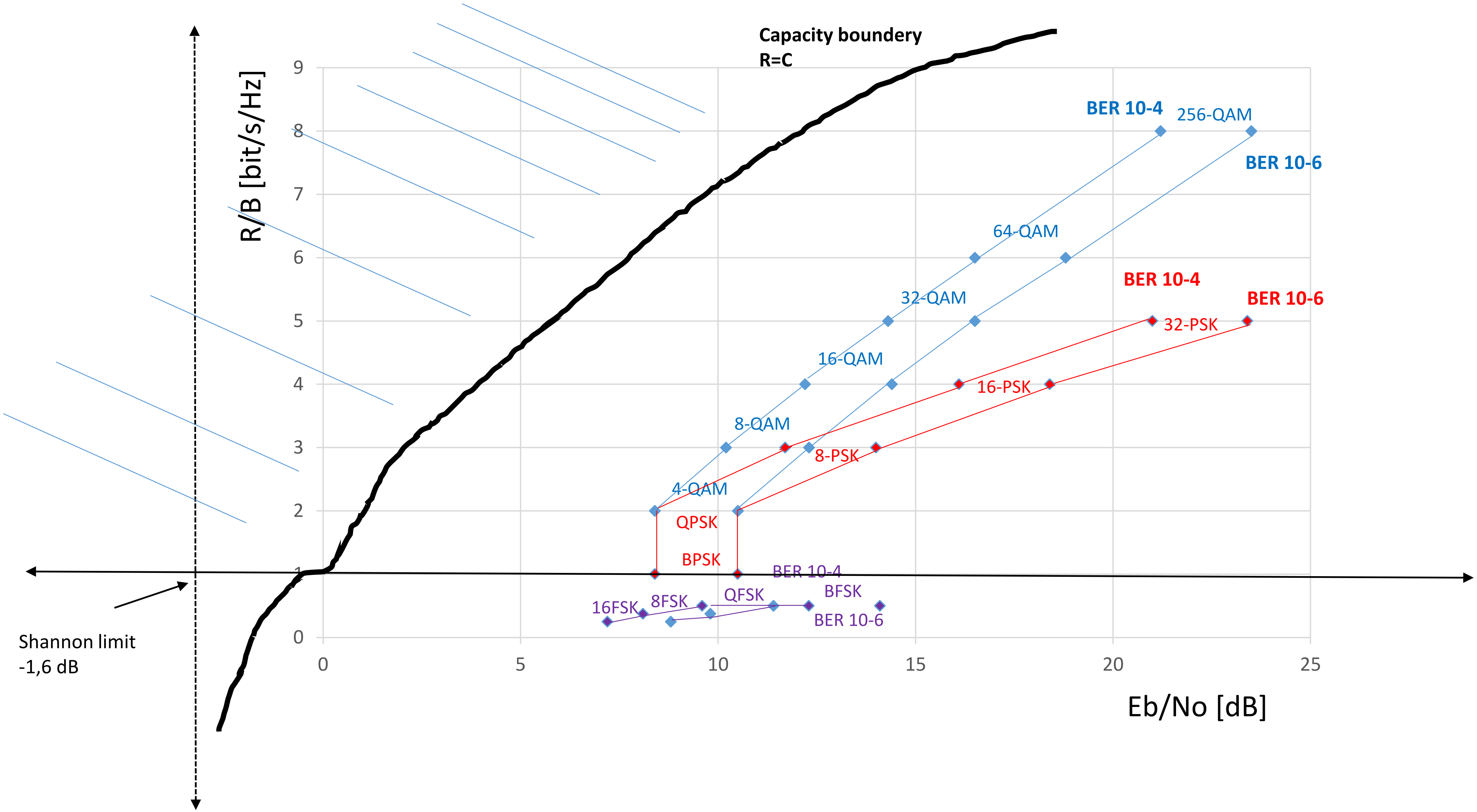
Example 2:
2 – PSK, PAM binary antipodal

$$P_{be} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$10^{-6} = Q\ (4{,}7534)$
Eb/N0 = (4,7534)^2/2
Eb/N0 = 10log(Eb/N0)=10,5 dB

Krivka pre modulaciu sa pre lepsie P_{be} posuva doprava (potvrdenie teorie)!!!

BANDWIDTH-EFFICIENCY PLANE



Spectral Efficiency (1/2)

The Spectral Efficiency (measured in b/s/Hz) of a modulation scheme with transmission rate R and bandwidth B is defined as

$$\rho = R/B$$

Consider an M-ary modulation scheme with baseband pulse $s(t)$ with duration T and null-to-null bandwidth $2/T$, then $R = (1/T)\log_2(M)$

	Modulation Format	Bandwidth	Spectral Efficiency
	ASK (DSB)	$2/T$	$(1/2)\log_2(M)$
	ASK (SSB)	$1/T$	$\log_2(M)$
	PSK	$2/T$	$(1/2)\log_2(M)$
	QAM (M)	$2/T$	$(1/2)\log_2(M)$
	QAM (M^2)	$2/T$	$\log_2(M)$
coherent	FSK $(\Delta_f = 1/2T)$	$(M+3)/2T$	$(2/(M+3))\log_2(M)$
noncoherent	FSK $(\Delta_f = 1/T)$	$(M+1)/T$	$(1/(M+1))\log_2(M)$
noncoherent	FSK	M / T	$log_2(M)/M$

For M-ary FSK, the frequency separation between adjacent frequencies is Δf = $1/T_s$ for signal orthogonality.

The bandwidth required for M signals is B = M Δf = M/T_s.

Spectral Efficiency (2/2)

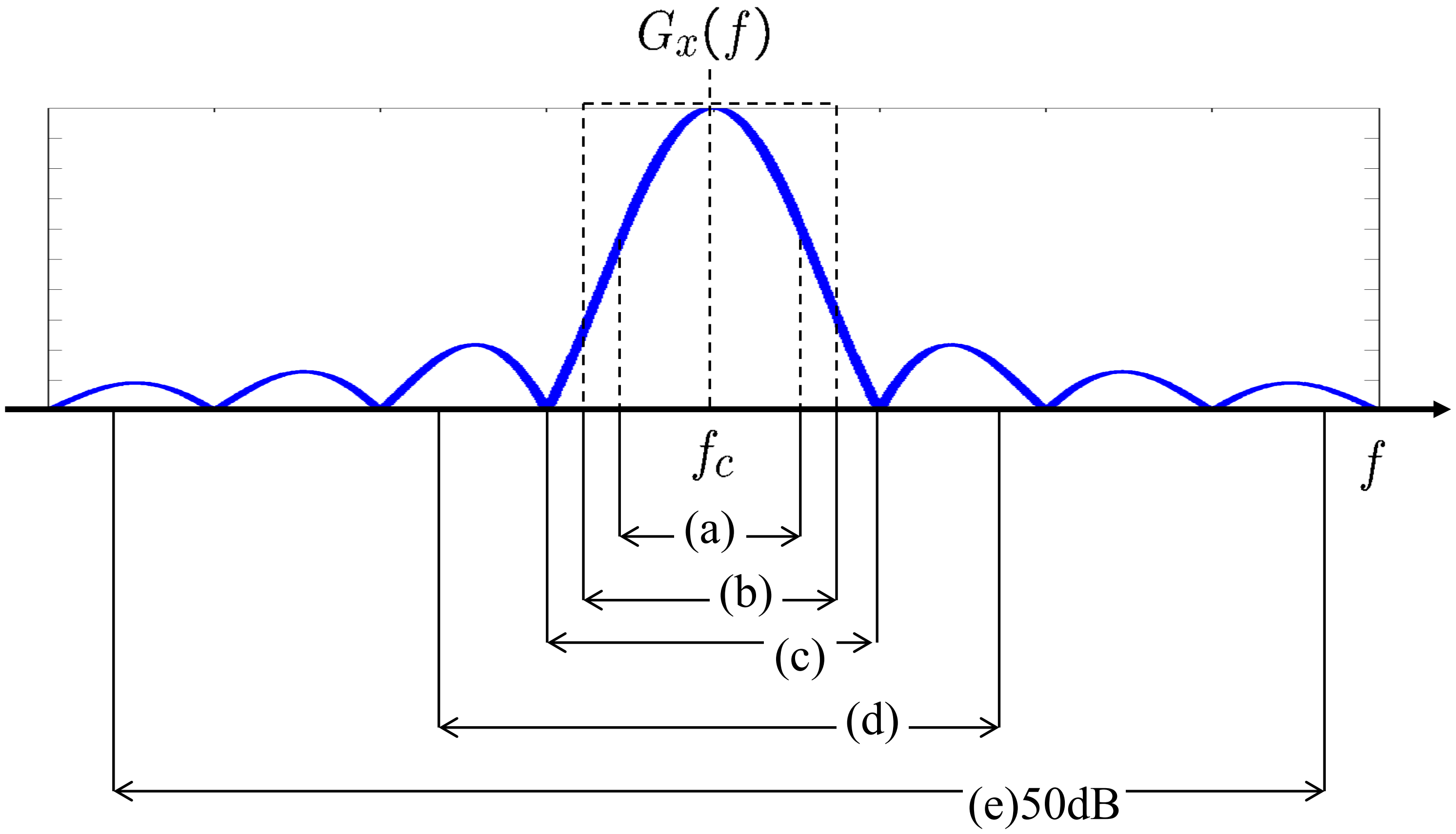
A different bandwidth definition scales the spectral efficiency

Halving null-to-null bandwidth (almost as 3dB bandwidth):

	Modulation Format	Bandwidth	Spectral Efficiency
	ASK (DSB)	$1/T$	$\log_2(M)$
	ASK (SSB)	$1/2T$	$2\log_2(M)$
	PSK	$1/T$	$\log_2(M)$
	QAM (M)	$1/T$	$\log_2(M)$
	QAM (M^2)	$1/T$	$2\log_2(M)$
coherent	FSK ($\Delta_f = 1/2T$)	$(M+3)/4T$	$(4/(M+3))\log_2(M)$
noncoherent	FSK ($\Delta_f = 1/T$)	$(M+1)/2T$	$(2/(M+1))\log_2(M)$
noncoherent	FSK	M / T	$log_2(M)/M$

a) Half-power bandwidth
b) Noise equivalent bandwidth
c) Null-to-null bandwidth
d) Fractional power containment bandwidth
e) Bounded power spectral density
f) Absolute bandwidth

(a) Half-power bandwidth. This is the interval between frequencies at which Gx(f) has dropped to half-power, or 3 dB below the peak value.
(c) Null-to-null bandwidth. The most popular measure of bandwidth for digital communications is the width of the main spectral lobe, where most of the signal power is contained.



shaping and is independent of M. For unfiltered MPSK (i.e. MPSK with rectangular pulses) the nominal (i.e. main lobe null-to-null) bandwidth is $2/T_o$ Hz. In this case the spectral efficiency (given by equation (11.36(b)), is:

$$\eta_s = 0.5 \log_2 M \quad \text{(bit/s/Hz)} \tag{11.42(a)}$$

The maximum possible, ISI-free, spectral efficiency occurs when pulse shaping is such that signalling takes place in the double sided Nyquist bandwidth $B = 1/T_o$ Hz, i.e.:

$$\eta_s = \log_2 M \quad \text{(bit/s/Hz)} \tag{11.42(b)}$$

Thus we usually say BPSK has an efficiency of 1 bit/s/Hz and 16-PSK 4 bit/s/Hz. Table 11.4 compares the performance of several PSK systems.

Table 11.4 *Comparison of several PSK modulation techniques.*

	Required E_b/N_0 for $P_b = 10^{-6}$	*Minimum channel bandwidth for ISI-free signalling (R_b = bit rate)*	*Max spectral efficiency (bit/s/Hz)*	*Required CNR in minimum channel bandwidth*
PRK	10.6 dB	R_b	1	10.6 dB
QPSK	10.6 dB	$0.5R_b$	2	13.6 dB
8-PSK	14.0 dB	$0.33R_b$	3	18.8 dB
16-PSK	18.3 dB	$0.25R_b$	4	24.3 dB