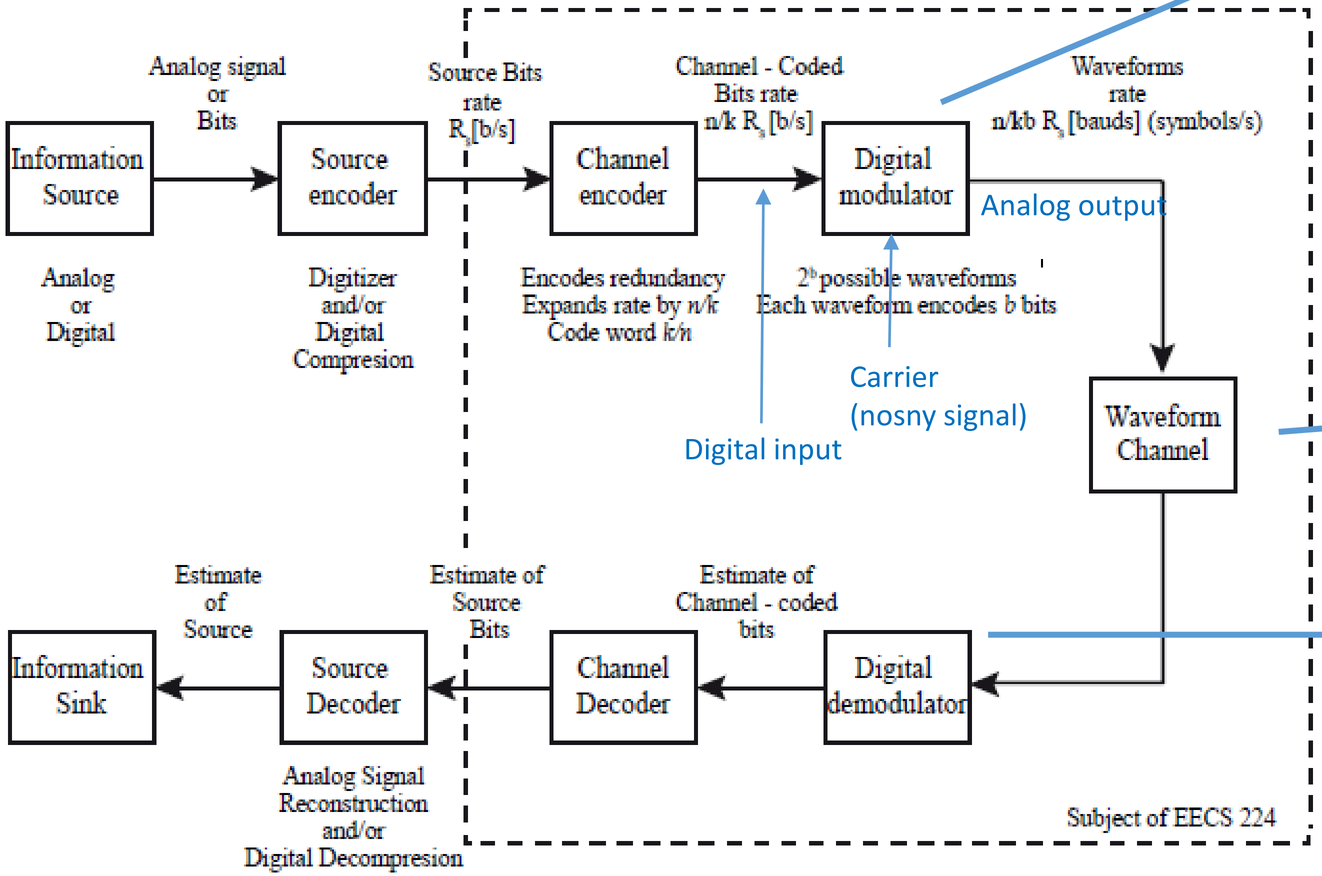
- Interface to the communication channel
- To map binary info sequence into signal waveform
- Binary modulation $0 \rightarrow s_0(t)$, $1 \rightarrow s_1(t)$
- M-ary modulation $M=2^b$ distinct waveforms $s_i(t)$,



- The physical medium
- Additive (thermal) noise

AWGN channel (Additive White Gaussian Noise)

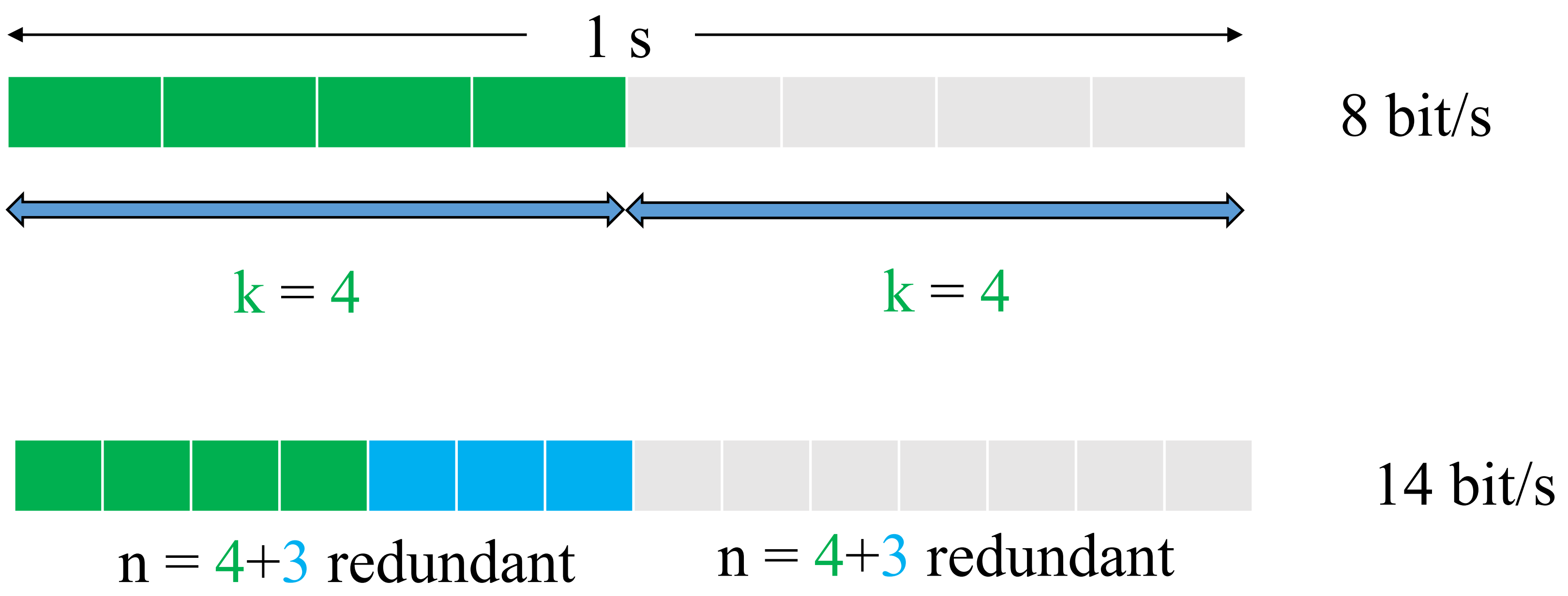
- Average probability of a bit-error at the output of the decoder

Figure 1.1 Basic elements of a digital communication system

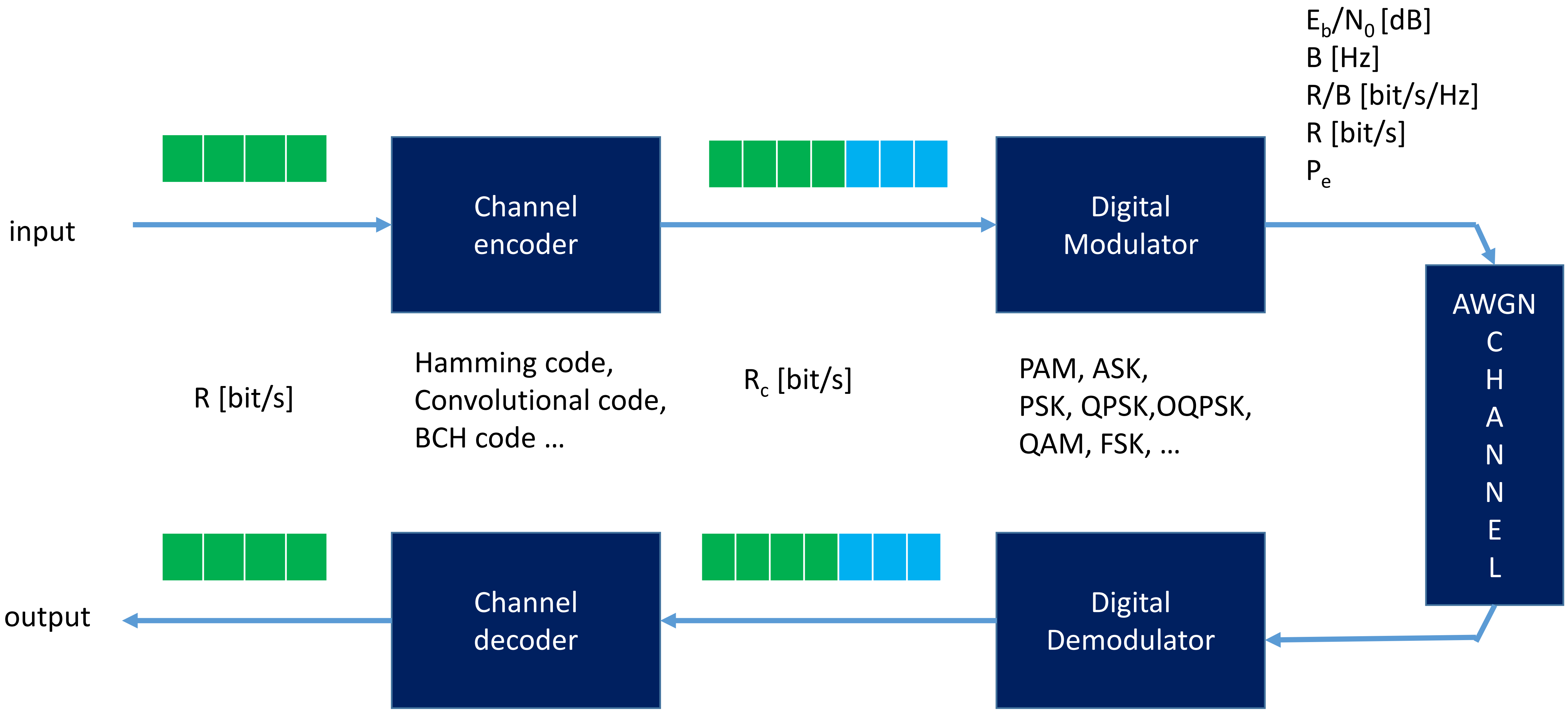
Objasnenie vztahu $R_c=(n/k)R$:

n = celkovy pocet bitov
k = pocet informacnych bitov
r = pocet redundantnych bitov

R=8 bit/s, kod (7,4), n=7, k=4, r=3



$$R_c = (n/k)\,R = (7/4)\,8 = 14 \text{ bit/s}$$


E_b/N_0 [dB]
B [Hz]
R/B [bit/s/Hz]
R [bit/s]
P_e
input
Channel encoder
Digital Modulator
AWGN CHANNEL
R [bit/s]
Hamming code, Convolutional code, BCH code …
R_c [bit/s]
PAM, ASK, PSK, QPSK,OQPSK, QAM, FSK, …
output
Channel decoder
Digital Demodulator

Hamming code (n,k) (7,4) – dokaze opravit jednoduche chyby,
t.j. chybu v jednom bite v postupnosti 7 bitov

The code generator matrix $\mathbf{G}$ and the parity-check matrix $\mathbf{H}$ are:

$$\mathbf{G} := \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}_{4,7}$$

and

$$\mathbf{H} := \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}_{3,7}.$$

Zakodujme postupnost bitov: 1 1 0 0

Spocitame 1 a 2 riadok G matice modulo 2

Zakodovana postupnost bitov: 1 1 0 0 0 1 1

Dekodujme prijatu postupnost bitov : 1 1 0 0 0 1 1

Spocitame 1, 2, 6, 7 stlpec H matice

Syndrom: 0 0 0 znamena prenos bez chyby

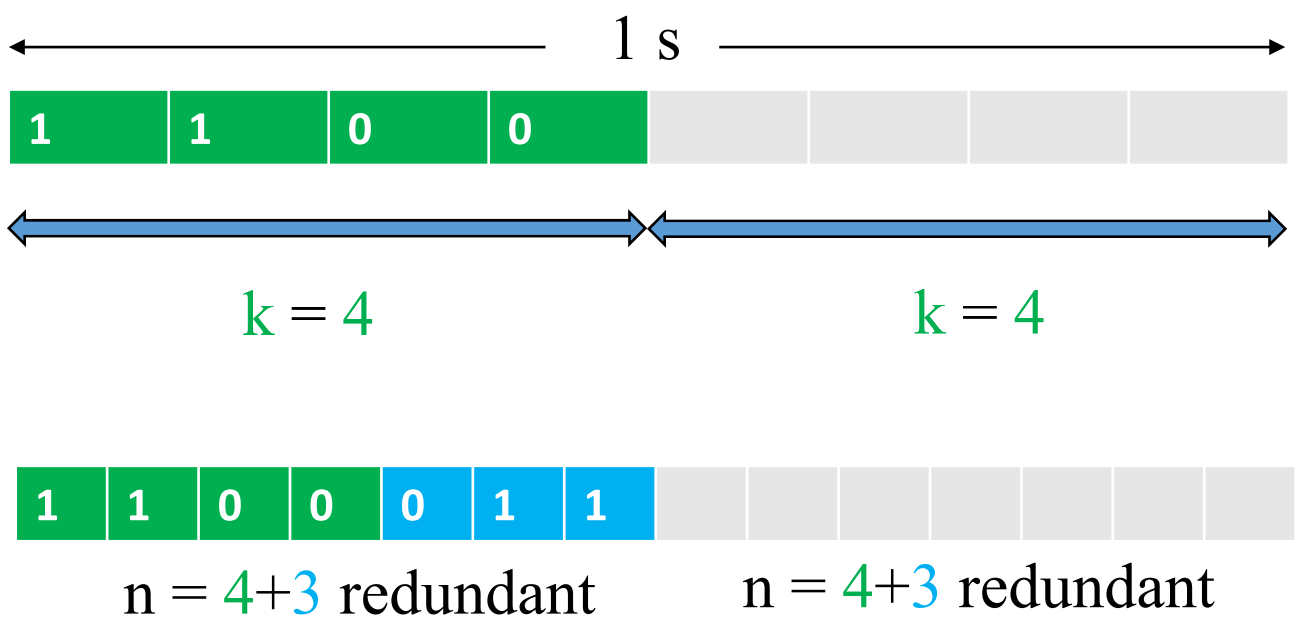
Dekodujme prijatu postupnost s chybou : 1 1 0 1 0 1 1

Spocitajme 1, 2, 4, 6, 7 stlpec H matice

Syndrom: 1 1 1 sa rovna 4. stlpcu H matice – chyba na 4. mieste

Modulo 2:

0+0=0
0+1=1
1+0=1
1+1=0



Zakodujme postupnost bitov: 1 1 0 0

Spocitame 1 a 2 riadok G matice modulo 2

Zakodovana postupnost bitov: 1 1 0 0 0 1 1

Dekodujme prijatu postupnost bitov : 1 1 0 0 0 1 1

Spocitame 1, 2, 6, 7 stlpec H matice

Syndrom: 0 0 0 znamena prenos bez chyby

1	1	0	1	0	1	1	1	1	0	0	0	1	0

Dekodujme prijatu postupnost s chybou:
1 1 0 1 0 1 1

Spocitame 1, 2, 4, 6, 7 stlpec H matice

Syndrom: 1 1 1 sa rovna 4. stlpcu H matice
– chyba na 4. mieste (1 zmenit na 0)

Dekodujme prijatu postupnost s chybou:
1 1 0 0 0 1 0

Spocitame 1, 2, 6 stlpec H matice

Syndrom: 0 0 1 sa rovna 7. stlpcu H matice
– chyba na 7. mieste (0 zmenit na 1)

The code generator matrix $\mathbf{G}$ and the parity-check matrix $\mathbf{H}$ are:

$$\mathbf{G} := \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}_{4,7}$$

and

$$\mathbf{H} := \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}_{3,7}.$$

8 WHY USE ERROR-CORRECTION CODING

Error-correction coding can be regarded as a vehicle for effecting various system trade-offs.

Figure 8.1compares two curves depicting bit-error performance versus Eb/No.

One curve represents a typical modulation scheme without coding. The second curve represents the same modulation with coding. Demonstrated below are four benefits or trade-offs that can be achieved with the use of channel
coding.

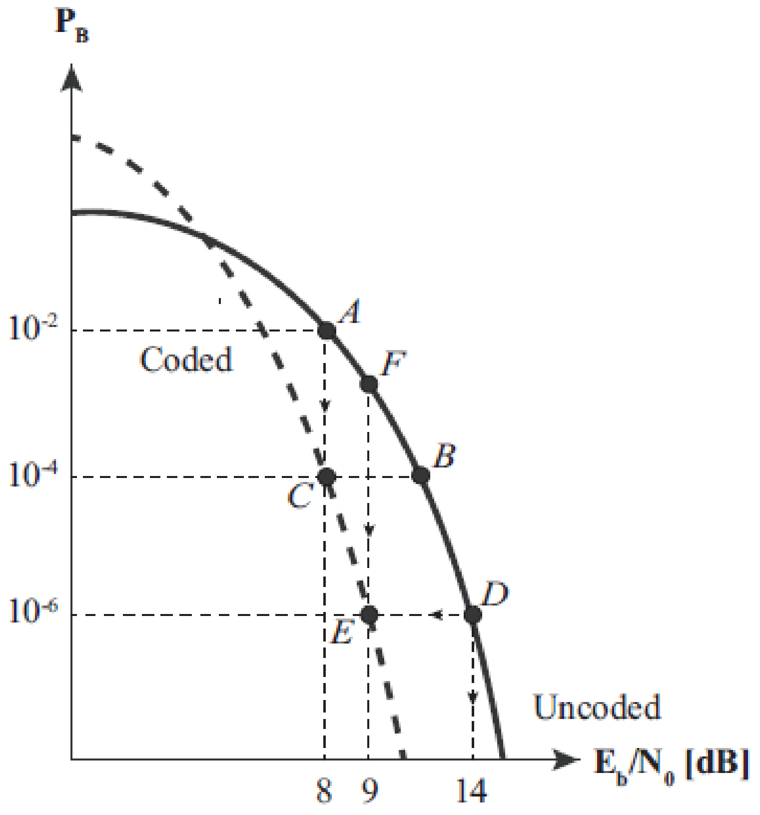


Figure 8.1 Comparison of typical coded versus uncoded error performance.

8.1 TRADE-OFF 1: ERROR PERFORMANCE VERSUS BANDWIDTH

Imagine that a simple, inexpensive voice communication system has just been developed and delivered to a customer. The system does not use error-correction coding. Consider that the operating point of the system can be depicted by point A in Figure 8.1 $\left(\frac{E_b}{N_0} = 8dB \text{ and } P_B = 10^{-2}\right)$. After a few trials, there are complaints about the voice quality: the customer suggests that the bit-error probability should be lowered to 10^{-4}. The usual way of obtaining better error performance in such a system would be by effecting an operating point movement from point A to, say, point B in Figure 8.1. However, suppose that the $\frac{E_b}{N_0}$ of 8 dB is the most that is available in this system. The figure suggests that one possible trade-off is to move the operating point from point A to point C. That is, "walking" down the vertical line to point C on the coded curve can provide the customer with improved error performance. What does it cost? Aside from the new components (encoder and decoder) needed, the price is more transmission bandwidth. Error-correction coding needs redundancy. If we assume that the system is a real-time communication system (such that the message may not be delayed), the addition of redundant bits dictates a faster rate of transmission, which of course means more bandwidth.

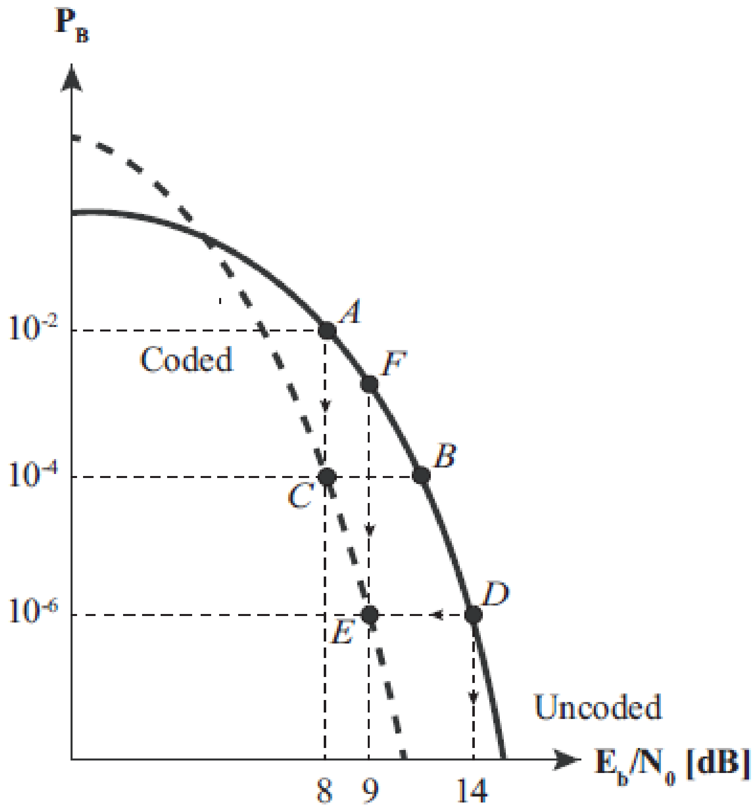


Figure 8.1 Comparison of typical coded versus uncoded error performance.

8.2 TRADE-OFF 2: POWER VERSUS BANDWIDTH

Consider that a system without coding, operating at point D in Figure 6.9 $\left(\frac{E_b}{N_0} = 14dB \text{ and } P_B = 10^{-6}\right)$, has been delive·red to a customer. The customer has no complaints about the quality of the data, but the equipment is having some reliability problems as a result of providing an $\frac{E_b}{N_0}$ of 14 dB. In other words, the equipment keeps breaking down. If the requirement on $\frac{E_b}{N_0}$ or power could be reduced, the reliability difficulties might also be reduced. Figure 8.1 suggests a trade-off by moving the operating point from point D to point E. That is, if error-correction coding is introduced, a reduction in the required $\frac{E_b}{N_0}$ can be achieved. Thus, the trade-off is one in which the same quality of data is achieved, but the coding allows for a reduction in power or $\frac{E_b}{N_0}$. What is the cost? The same as before - more bandwidth.

Notice that for *non-real-time* communication systems, error-correction coding can he used with a somewhat different trade-off. It is possible to obtain improved bit-error probability or reduced power (similar to trade-off 1 or 2 above) by paying the price of delay instead of bandwidth.

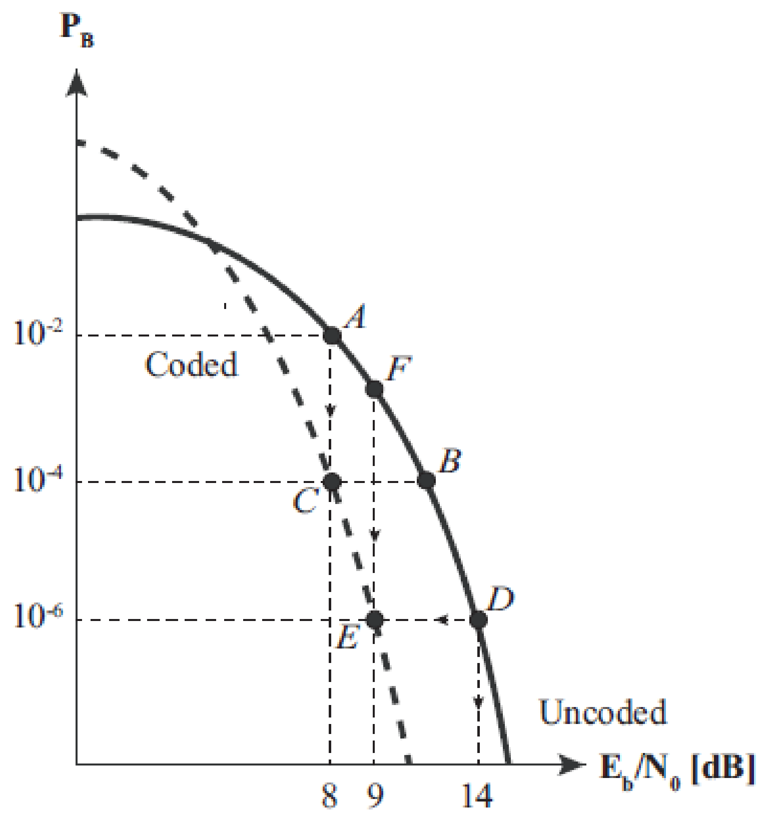


Figure 8.1 Comparison of typical coded versus uncoded error performance.

8.3 Coding Gain

The trade-off example described in the previous section has allowed a reduction in $\frac{E_b}{N_0}$ from 14 dB to 9 dB, while maintaining the same error performance. In the context of this example and Figure 8.1, we now define *coding gain*. For a given bit-error probability, coding gain is defined as the "relief" or reduction in $\frac{E_b}{N_0}$ that can be realized through the use of the code. Coding gain G is generally expressed in dB, such as

$$G[dB] = \left(\frac{E_b}{N_0}\right)_u [dB] - \left(\frac{E_b}{N_0}\right)_c [dB] \quad (8.1)$$

where $\left(\frac{E_b}{N_0}\right)_u$ and $\left(\frac{E_b}{N_0}\right)_c$, present the required $\frac{E_b}{N_0}$, uncoded and coded, respectively.

8.4 Trade-Off 3: Data Rate versus Bandwidth

Consider that a system without coding, operating at point D in Figure 8.1 $\left(\frac{E_b}{N_0} = 14dB \text{ and } P_B = 10^{-6}\right)$ has been developed. Assume that there is no problem with the data quality and no particular need to reduce power. However, in this example, suppose that the customer's data rate requirement increases. Recall the relationship:

$$\frac{E_b}{N_0} = \frac{P_r}{N_0}\left(\frac{1}{R}\right) \quad (8.2)$$

If we do nothing to the system except increase the data rate R, the above expression shows that the received $\frac{E_b}{N_0}$ would decrease, and in Figure 8.1, the operating point would move upwards from point D to, let us say, some point F. Now, envision "walking" down the vertical line to point E on the curve that represents coded modulation. Increasing the data rate has degraded the quality of the data. But, the use of error-correction coding brings back the same quality at the same power level $\left(\frac{P_r}{N_0}\right)$. The $\frac{E_b}{N_0}$ is reduced, but the code facilitates getting the same error probability with a lower $\frac{E_b}{N_0}$. What price do we pay for getting this higher data rate or greater capacity? The same as before- increased bandwidth.

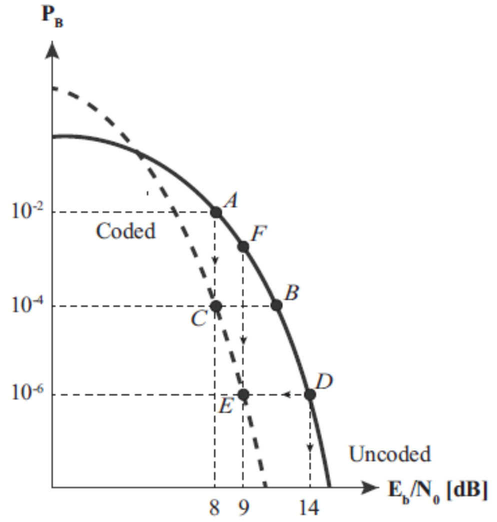


Figure 8.1 Comparison of typical coded versus uncoded error performance.

8.5 Trade-Off 4: Capacity versus Bandwidth

Trade-off 4 is similar to trade-off 3 because both achieve increased capacity. A spread-spectrum multiple access technique, called code-division multiple access (CDMA), is one of the schemes used in cellular telephony. In CDMA, where users simultaneously share the same spectrum, each user is an interferer to each of the other users in the same cell or nearby cells. Hence, the capacity (maximum number of users) per cell is inversely proportional to $\frac{E_b}{N_0}$. In this application, a lowered $\frac{E_b}{N_0}$ results in a raised capacity; the code achieves a reduction in each user's power, which in turn allows for an increase in the number of users. Again, the cost is more bandwidth. But, in this case, the signal-bandwidth expansion due to the error-correcting code is small compared with the more significant spread-spectrum bandwidth expansion, and thus, there is no impact on the transmission bandwidth.

In each of the above trade-off examples, a "traditional" code involving redundant bits and faster signaling (for a real-time communication system) has been assumed: hence, in each case, the cost was expanded bandwidth. However, there·, exists an error-correcting technique, called *trellis-coded* modulation, that does not require faster signaling or expanded bandwidth for real-time systems.

8.6 CODE PERFORMANCE AT LOW VALUES OF E_B/N_0

The reader is urged to solve Exercise 2, with is similar to Problem 1. In part a) of Exercise 2, where an $\frac{E_b}{N_0}$ of 14 dB is given, the result is a message - error performance improvement through the use of coding. However, in part b) of Exercise 2, where the $\frac{E_b}{N_0}$ has been reduced to 10 dB, coding provides no improvement, in fact, there is a degradation. One might ask, why does part b) of Exercise 2 manifest a degradation? After all, the same procedure is used for applying the code in both parts of the problem. The answer can be seen in the coded-versus-uncoded pictorial shown in Figure 8.1, Even though Exercise 2 deals with message-error probability, And Figure 8.1 displays bit-error probability, the following explanation still applies. In all such plots, there is a crossover between the curves (usually at some low value of $\frac{E_b}{N_0}$). The reason for such crossover (threshold) is that every code system has some fixed error-correcting capability. If there are more errors within a block than the code is capable of correcting, the system will perform poorly. Imagine that $\frac{E_b}{N_0}$ is continually reduced. What happens at the output of the demodulator? It makes more and more errors. Therefore. such a continual decrease in $\frac{E_b}{N_0}$ must eventually cause some threshold to be reached where the decoder becomes overwhelmed with errors. When that threshold is crossed, we can interpret the degraded performance as being caused by the redundant bits consuming energy but giving block nothing beneficial in return.

Does it strike the reader as a paradox that operating in a region (low values of $\frac{E_b}{N_0}$), where one would best like to see an error-performance improvement, is where the code makes things worse? There is, however, a class of powerful codes called turbo codes that provide error -performance improvement, at low values of $\frac{E_b}{N_0}$: the crossover point is lower for turbo codes compared with conventional codes.

8.7 SOLVED PROBLEM

Problem 1

Compare the message error probability for a communications link with and without the use of error-correction coding. Assume that the uncoded transmission characteristics are: BPSK modulation. Gaussian noisse $\frac{P_r}{N_0} = 43{,}776$, data rate $R = 4800\ bits/s$. For the coded case, also assume the use of a (15.11) error-correcting code that is capable of correcting any single-error pattern within a block of 15 bits. Consider that the demodulator makes hard decisions and thus feeds the demodulated code bits directly to the decoder, which in turn outputs an estimate of the original message.

Solution

Following Equation, let $p_u = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$ and $p_u = Q\left(\sqrt{\frac{2E_c}{N_0}}\right)$ be the uncoded and coded channel symbol error probabilities, respectively, where $\frac{E_b}{N_0}$ is the bit energy per noise spectral density and $\frac{E_c}{N_0}$ is the code-bit energy per noise spectral density.

Without coding

$$\frac{E_b}{N_0} = \frac{P_r}{N_0}\left(\frac{1}{R}\right) = 9{,}12\ (9{,}6dB)$$

And

$$p_u = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q(\sqrt{18{,}24}) = 1{,}02 * 10^{-5} \tag{8.3}$$

where the following approximation of Q(x) was used:

$$Q(x) \approx \frac{1}{x\sqrt{2\pi}} exp\left(\frac{-x^2}{2}\right), for\ x > 3$$

The probability that the uncoded message block P_M^u will be received in error is 1 minus the product of the probabilities that each bit will be detected correctly. Thus,

$$P_M^u = 1 - (1 - p_u)^k$$
$$= 1 - \underbrace{(1 - p_u)^{11}}_{\substack{probability\ that\ all \\ 11 bits\ in\ uncoded \\ block\ are\ correct}} = \underbrace{1{,}12 * 10^{-4}}_{\substack{probability\ that\ at \\ least\ 1\ bit\ out \\ 11\ is\ in\ error}} \tag{8.4}$$

Without coding

Assuming a real-time communication system such that delay is unacceptable, the channel-symbol rate or code-hit rate R, is 15/11 times the data hit rate:

$$R_c = 4800 * \frac{15}{11} \approx 6545\ bps$$

And

$$\frac{E_c}{N_0} = \frac{P_r}{N_0}\left(\frac{1}{R_c}\right) = 6{,}69\ (8{,}3\ dB)$$

The $\frac{E_c}{N_0}$ for each colic bit is less than that for the data bit in the uncoded case because the channel-bit has increased, but the transmitter power is assumed to be fixed:

$$p_c = Q\left(\sqrt{\frac{2E_c}{N_0}}\right) = Q\left(\sqrt{13{,}38}\right) = 1{,}36 * 10^{-4} \qquad (8.5)$$

It can be seen by comparing the results of Equation 8.3 with those of Equation 8.5 that because redundancy was added, the channel bit-error probability has degraded. More bits must he detected during the same time interval and with the same available power: the performance improvement due to the coding is not yet apparent. We now compute the coded message error rate P_M^c:

$$p_M^c = \sum_{j=2}^{n=15} \binom{15}{j} (p_c)^j (1 - p_c)^{15-j}$$

The summation is started with $j = 2$, since the code corrects all single errors within a block of n = 15 bits. An approximation is obtained by using only the first term of the summation. For p_c, we use the value calculated in Equation 8.5:

$$P_M^c = \binom{15}{2} (p_c)^2 (1 - p_c)^{13} = 1{,}94 * 10^{-6} \qquad (8.6)$$

By comparing the results of Equation 8.4 with 8.6, we can see that the probability of message error has improved by a factor of 58 due to the error-correcting code used in this example. This example illustrates the typical behavior of all such real-time communication systems using error-correction coding. Added redundancy means faster signaling, less energy per channel symbol, and more errors out of the demodulator. The benefits arise because the behavior of the decoder will (al reasonable values of $\frac{E_b}{N_0}$) more than compensate for the poor performance of the demodulator.